### 4A Review of index laws

You learned in earlier years that $2^3$ means ‘multiply two by itself three times’, that is, $2 \times 2 \times 2 = 8$. The index indicates the number of times the base is multiplied by itself.

In general,

\[
\begin{align*}
\text{Index form} & \quad \frac{3^4}{\text{Expanded form}} \quad 3 \times 3 \times 3 \times 3 = 81 \\
& \quad \text{Basic numeral} \quad a^m = a \times a \times \ldots \times a \text{, } m \text{ times}
\end{align*}
\]

In summary, the index laws are:

- **Multiplication:**
  \[a^m \times a^n = a^{m+n}\]

- **Division:**
  \[\frac{a^m}{a^n} = a^{m-n}\]

- **Raising to a power:**
  \[(a^m)^n = a^{mn}\]

- **Raising to the power of zero:**
  \[a^0 = 1\]

- **Raising to the power of one:**
  \[a^1 = a\]

- **Products and quotients:**
  \[(a \times b)^m = a^m \times b^m, \quad \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}\]

- **Negative powers:**
  \[a^{-m} = \frac{1}{a^m}\]

- **Fractional powers:**
  \[\frac{m}{a} = \sqrt[m]{a^m} = (\sqrt[m]{a})^m\]

When simplifying expressions and equations with indices, we generally use a combination of these laws.
WORKED EXAMPLE 1

Simplify \( \frac{3x^4y^3 \times 2x^2y^2z}{5x^8yz} \).

THINK
1. Add the indices of \( x \) and the indices of \( y \) and multiply the constants together in the numerator.

WRITE
\[
\frac{3x^4y^3 \times 2x^2y^2z}{5x^8yz} = \frac{6x^6y^5z}{5x^8yz} = \frac{6x^{-2}y^4z^0}{5x^2}
\]

2. Subtract the indices of \( x \), \( y \) and \( z \) in the denominator from those in the numerator.

3. Simplify to write the answer with positive indices and use the rule \( z^0 = 1 \).

When numbers with an index are then raised to another index, the indices are multiplied.

WORKED EXAMPLE 2

Simplify:
\[ \begin{align*}
&\text{a} \quad x^{-3}y^2 \times (2x^2y^{-1})^3 \\
&\text{b} \quad (a^4b^3)^{-2} + \frac{a^7b^2}{a^3b^{-4}}.
\end{align*} \]

THINK
\[ \begin{align*}
&\text{a} \quad \text{Remove the brackets by multiplying the indices.} \\
&\text{2. Add the indices of } x \text{ and } y. \\
&\text{3. Simplify to write the answer with a positive index.}
\end{align*} \]

WRITE
\[ \begin{align*}
&\text{a} \quad x^{-3}y^2 \times (2x^2y^{-1})^3 = x^{-3}y^2 \times 8x^6y^{-3} = 8xy^{-1} \\
&\text{= } \frac{8x^3}{y} \\
&\text{b} \quad (a^4b^3)^{-2} + \frac{a^7b^2}{a^3b^{-4}} = a^{-8}b^{-6} + \frac{a^7b^2}{a^3b^{-4}} \\
&\text{= } \frac{a^{-8}b^{-6}}{1} \times \frac{a^7b^{-4}}{a^3b^2} = \frac{a^{-5}b^{-10}}{a^7b^2} \\
&\text{= } \frac{a^{-12}b^{-8}}{a^{12}b^8} = \frac{1}{a^{24}b^{16}}
\end{align*} \]

WORKED EXAMPLE 3

Simplify:
\[ \begin{align*}
&\text{a} \quad 2^{n-1} \times 6^{2n} \times 3^{n+1} \\
&\text{b} \quad 2^n \times 4^{1-n} \times 16^{2n-1}.
\end{align*} \]

THINK
\[ \begin{align*}
&\text{a} \quad \text{Change the 6 into } 2 \times 3. \\
&\text{2. Remove the brackets by multiplying the indices.} \\
&\text{3. Add indices of numbers with base 2 and add indices of numbers with base 3.}
\end{align*} \]

WRITE
\[ \begin{align*}
&\text{a} \quad 2^{n-1} \times 6^{2n} \times 3^{n+1} = 2^{n-1} \times (2 \times 3)^{2n} \times 3^{n+1} \\
&\text{= } 2^{n-1} \times 2^{2n} \times 3^{2n} \times 3^{n+1} \\
&\text{= } 2^{3n-1} \times 3^{3n+1}
\end{align*} \]

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b 1 Change all numbers to a base of 2.
2 Remove the brackets by multiplying the indices.
3 Simplify by adding indices of numbers with base 2.

WORKED EXAMPLE 4

Simplify each of the following, expressing the answer with a positive index.

a \( \sqrt[4]{128} \times \sqrt[4]{64} \)

b \( \sqrt[3]{x^2y^6} + \sqrt[3]{x^3y^5} \)

THINK

a 1 Write the expression.
2 Write using fractional indices.
3 Write 128 and 64 in index form with a base of 2.
4 Multiply the powers.
5 Simplify and write the answer.

b 1 Write the expression.
2 Write the expression using index notation.
3 Remove the brackets by multiplying the powers.
4 Collect terms with the same base by subtracting the powers.
5 Simplify the powers.
6 Rewrite with positive powers.

Exercise 4A Review of index laws

1 Simplify the following expressions.

a \( \frac{12a^2b}{3a} \)

b \( \frac{8a^3b^4}{2ab} \)

c \( \frac{-a^2b^{-4}}{2ab^3} \)

d \( \frac{-3a^3b^2}{6a^{-2}b} \)

e \( \frac{3}{2a^2b^4} \times 3a^2b^{-1} \)

f \( \frac{1}{83}a \frac{1}{3}b^3 + 4a^{-2}b^{\frac{5}{2}} \)
2 **WE2** Simplify the following expressions.

\[ \frac{(x^3y^2)^2}{2x^4y^2} \quad \frac{(2xy^2)^3}{4x^2y^3} \quad \frac{(3x^3y^2)^2}{4(3x^2y^2)^4} \]

\[ \frac{2(x^2y^2)^3}{(2x^3y^3)} \quad \frac{(m^3n^2)^2}{2mn} \quad \frac{(2m^{-1}n^{-2})^{-3}}{2(m^2n^{-2})^4} \]

\[ \frac{(5m^2n^2)^2}{(3m^{-2}n^{-3})^2} \times \frac{3m^{-1}}{n^2} \quad \frac{(-2v^{-3}w^3)^3}{(5v^{-2}w^1)^{2}} \times \frac{2w^2}{(vw)^2} \quad \frac{(4v^{-1}w^2)^2}{(-3m^{-3}n^2)^{3}} \times \frac{2w^2}{(v^2-w^{-2})^2} \]

\[ \frac{(-3v^1w^2)^{-3}}{(2v^4w^{-1})^2} + \frac{9w^2}{(-2v^3)^2} \]

3 **MC** Expressing \( \sqrt[3]{ab} \) in index form gives:

- A \( ab^\frac{3}{2} \)
- B \( ab^{-3} \)
- C \( (ab)^{-\frac{3}{2}} \)

4 **WE3** Simplify each of the following.

\[ 2n^{-1} \times 4^n + 1 \times 16^n \]

\[ d \]

\[ \frac{27}{16} + \frac{4^n+2}{3n-1} \]

\[ c \]

\[ 6^n \quad \frac{5n^{-3} \times 3^n + 1}{75^n} \]

\[ f \]

\[ 72m^{-2} \times 4 \times 32m \]

5 **WE4** Simplify each of the following, expressing your answer with positive indices.

\[ \sqrt{9} \times \sqrt[3]{81} \]

\[ d \]

\[ \frac{3}{\sqrt{2}}(xy) + \sqrt{(x^2) y} \]

\[ g \]

\[ \frac{2}{\sqrt{x} + 1} \]

\[ j \]

\[ \frac{2}{\sqrt{x} + 1} \]

\[ k \]

\[ \frac{x}{\sqrt{x} + 2} + \frac{x}{\sqrt{x} + 2} \]

\[ l \]

\[ (y-4)\sqrt{y-4} \]

4B **Standard form and significant figures**

**Standard form**

Standard form (or scientific notation) involves a practical use of indices. A very large or very small number can be expressed in standard form as a more convenient way of writing it. This notation involves expressing the number as a number between one and ten multiplied by a power of 10.

\[ a \times 10^n \], where \( 1 \leq a < 10 \)

A computer may complete a basic operation in approximately 0.000 000 000 8 seconds. It is easier to write a number such as this in standard form as \( 8 \times 10^{-10} \) seconds.

Likewise, a light year is a measure of distance equal to 9 460 528 400 000 000 kilometres. In standard form, this number can be expressed as \( 9.460 \times 10^{15} \) seconds.

Standard form is not only a more economical means of expressing these numbers, it also makes calculations involving these numbers easier through the use of index laws.

To write a number in standard form:
- Move the decimal point so that the number appears to be between 1 and 10.
- Count the number of decimal places the decimal point has been moved (positive if moving left or negative if moving right).
- Multiply by the power of 10 equal to this number.
- The sign of the power will be positive if the magnitude of the original number is greater than 10.
- The sign of the power will be negative if the magnitude of the original number is between 0 and 1.
**WORKED EXAMPLE 5**

Solve \( \frac{350 000 \times 0.04}{70} \) and express as a basic numeral.

**THINK**

1. Express the problem in standard form.

\[
350 000 \times 0.04 \quad \text{and} \quad 70 \\
= 3.5 \times 10^5 \times 4 \times 10^{-2} \\
= 7 \times 10 \\
\]

2. Simplify the numerator using index laws where possible.

\[
14 \times 10^3 \\
= 2 \times 10^2 \\
\]

3. Divide using index laws where possible.

\[
= 2 \times 10^2 \\
\]

4. Express as a basic numeral.

\[
= 200 \\
\]

**WORKED EXAMPLE 6**

State the number of significant figures in the following numbers.

a. 3.205 60  

**THINK**

a. Significant figures are counted from the first non-zero digit (1–9). There are two zeros after the decimal point that are to the right of a non-zero digit, so all digits are significant.

**WRITE**

a. 3.205 60 has 6 significant figures.

b. 20.01  

**THINK**

b. Significant figures are counted from the first non-zero digit (1–9). All zeros between two non-zero digits are always significant. All digits in this case are significant.

**WRITE**

b. 20.01 has 4 significant figures.

c. 0.0034  

**THINK**

c. Significant figures are counted from the first non-zero digit (1–9). The first non-zero digit in this case is 3. Only 3 and 4 are significant.

**WRITE**

c. 0.0034 has 2 significant figures.

d. 35 000  

**THINK**

d. The trailing zeros at the end of a number are not considered significant. Only 3 and 5 are significant.

**WRITE**

d. 35 000 has 2 significant figures.

---

**Significant figures**

Often we will be interested in all the figures in a particular number.

- Significant figures are counted from the first non-zero digit (1–9). For example, 0.0092 has two significant figures (9 and 2).
- Any zeros at the end of the number after the decimal point are considered to be significant. For example, 0.250 has three significant figures (2, 5 and 0), whereas 0.025 has two significant figures (2 and 5).
- The trailing zeros at the end of a number are not considered significant. For example, 1200 has two significant figures (1 and 2).
- All zeros between two non-zero digits are always significant. For example, 102.587 has 6 significant figures (1, 0, 2, 5, 6 and 7).
Calculations involving significant figures

When performing calculations associated with significant figures, the following rules apply.
• When adding or subtracting numbers, count the number of decimal places to determine the number of significant figures. The answer cannot contain more places after the decimal point than the least number of decimal places in the numbers being added or subtracted.
• When multiplying or dividing numbers, count the number of significant figures. The answer cannot contain more significant figures than the number being multiplied or divided with the least number of significant figures.

WORKED EXAMPLE 7

Evaluate, expressing your answer to the appropriate number of significant figures:

\[
\begin{align*}
a & \quad 345.87 + 20.1 \\
b & \quad 23.020 \times 0.023.
\end{align*}
\]

**THINK**

**WRITE**

\[
\begin{align*}
a & \quad 345.87 \text{ has } 2 \text{ decimal places.} \\
& \quad 20.1 \text{ has } 1 \text{ decimal place.} \\
& \quad \text{The answer will have } 1 \text{ decimal place.} \\
2 & \quad \text{Add the numbers.} \\
& \quad 347.87 + 20.1 = 367.97 \\
3 & \quad \text{Round the answer to 1 decimal place.} \\
& \quad 368.0 \\
4 & \quad \text{Interpret this answer.} \\
& \quad \text{The answer has 1 decimal place and 4 significant figures.}
\end{align*}
\]

\[
\begin{align*}
b & \quad 23.020 \text{ has } 5 \text{ significant figures.} \\
& \quad 0.023 \text{ has } 2 \text{ significant figures.} \\
& \quad \text{The answer will have } 2 \text{ significant figures.} \\
2 & \quad \text{Multiply the numbers.} \\
& \quad 23.020 \times 0.023 = 0.52946 \\
3 & \quad \text{Express the answer to 2 significant figures.} \\
& \quad 0.53
\end{align*}
\]

Exercise 4B Standard form and significant figures

1. Express the following in standard form.
   \[
   \begin{align*}
a & \quad 360\,400 \\
b & \quad 0.0324 \\
c & \quad 1\,023.98 \\
d & \quad 213.457 \\
e & \quad 0.000\,100\,31 \\
f & \quad 570\,201\,009
\end{align*}
   \]

2. Solve by expressing the numbers in standard form and simplifying using index laws. Express your answer as a basic numeral.
   \[
   \begin{align*}
a & \quad \frac{28\,000}{350} \\
b & \quad \frac{420\,000}{1400} \\
c & \quad \frac{11\,200\,000}{2800} \\
d & \quad \frac{80\,000\,000}{16\,000} \\
e & \quad \frac{3\,100\,000}{1550} \\
f & \quad \frac{7\,500\,000}{1500} \\
g & \quad \frac{0.000\,24}{0.3} \\
h & \quad \frac{0.000\,018}{0.06} \\
i & \quad \frac{0.000\,056}{0.0350} \\
j & \quad \frac{0.000\,84}{0.0021} \\
k & \quad \frac{5\,800\,000}{0.02} \\
l & \quad \frac{130\,000}{0.0026} \\
m & \quad \frac{0.0066}{11\,000} \\
n & \quad \frac{0.000\,095}{190\,000} \\
o & \quad \frac{18\,000\times0.0045}{900} \\
p & \quad \frac{4900\times0.001\,75}{35} \\
q & \quad \frac{750\,000\,00\times0.000\,025}{1250} \\
r & \quad \frac{25\,600\,000\times0.000\,000\,004}{0.0064}
\end{align*}
   \]
3. **MC** 10.0673 expressed in standard form is:
   - A $1.00673 \times 10^1$
   - B $10.0673 \times 10^{-1}$
   - C $0.100673 \times 10^{-2}$
   - D $0.100673 \times 10^2$
   - E $1.00673 \times 10^{-1}$

4. **WE6** Specify the number of significant figures in the following.
   - a) 0.023
   - b) 10.21
   - c) 3045
   - d) 210.50
   - e) 0.120 10
   - f) 34 700.002
   - g) 0.100 673
   - h) 7620
   - i) 190.00
   - j) 0.000 002
   - k) 4730.90
   - l) 2 800 000

5. **WE7** Calculate the following to the correct number of significant figures.
   - a) $2.456 + 0.9$
   - b) $12.340 + 1.02$
   - c) $0.2507 - 0.120$
   - d) $1.903 \times 230.576$
   - e) $28.1 \times 2.1020$
   - f) $403.5 \div 5.1$
   - g) $2.01 \div 0.05080$

6. **MC** The solution to $130.70 - 28.9913$ with the correct number of significant figures is:
   - A $101.71$
   - B $101.7090$
   - C $101.7$
   - D $101$
   - E $101.70$

7. **MC** The solution to $32.3695 + 1.870$ with the correct number of significant figures is:
   - A $17.3$
   - B $17$
   - C $17.309$
   - D $17.31$
   - E $17.3100$

8. Complete the following calculations, expressing your answer to the appropriate number of significant figures.
   - a) It is 1.35 kilometres from Jane’s house to school. Her average step length is 0.7 metres. How many steps does it take for Jane to walk to school?
   - b) If a container of sugar cubes has a mass of 250 g when full (excluding the mass of the container), how many sugar cubes would be required to fill the container if they each have a mass of 3.24 g?

9. **MC** The outer ‘skin’ of a human cell, the cell membrane, is approximately $0.000 000 008 4$ metres thick. If the radius of the cell (including the cell membrane) is $0.000 004 2$ metres, what fraction of the radius does the cell membrane constitute?

10. **MC** An Olympic size swimming pool contains 2,500,000 litres of water. The average daily water usage for a family of four is 625 litres. How long would it take for a family to use a volume of water equivalent to an Olympic size pool?

11. **MC** The Earth has a mass of approximately 5970 yottagrams (where a yottagram, Yg, is $10^{21}$ kg). The mass of the Moon is 73 500 zettagrams (where a zettagram, Zg, is $10^{18}$ kg). What percentage of the mass of the Earth is the mass of the Moon?

---

**4C Transposition**

A formula is an equation or a rule that defines the relationship between two or more variables. If a formula describes a relationship between two variables, both of which are to the power of 1, and does not contain terms that include a product or quotient of those variables, then such a relation is said to be linear.

The graph that represents a linear relation is a straight line, which is how the term linear is derived.

For example:

\[ x - 4y - 7 = 0 \quad \text{and} \quad y = -3x + 6 \]

are linear relations, whereas

\[ x + y - xy = 3 \quad \text{or} \quad x^2 + y = 29 \quad \text{or} \quad \frac{x}{y} = 7 \]

are not (as explained previously).

Linear relations are often found in practical situations. For example, the formula for the circumference of a circle, \( C = \pi D \), and the formula for the conversion of temperature from degrees Celsius to degrees Fahrenheit, \( F = \frac{9}{5} C + 32 \), both describe linear relations. If we wanted to find many values of \( C \) given...
various values of $F$, it would be more convenient to have the corresponding formula — the formula that would have $C$ on one side and everything else on the other side of the equals sign. The variable that is by itself is called the subject of the formula (that is, a formula describes its subject in terms of all other variables). In the formula $I = 20R$, $I$ is the subject. To make $R$ the subject, we need to rearrange the formula. Such a rearrangement is called transposition.

To transpose the equation

$$I = 20R$$

divide both sides of the equation by 20.

$$\frac{I}{20} = \frac{20R}{20}$$

Simply.

$$\frac{I}{20} = R$$

Write the subject on the left-hand side.

$$R = \frac{I}{20}$$

To rearrange or transpose a formula, we need to perform the same inverse operations to both sides of the equation until the desired result is achieved.

**WORKED EXAMPLE 8**

Transpose the formula $4x = 2y - 3$ to make $y$ the subject.

**THINK**

1. Write the given formula.
   $$4x = 2y - 3$$

2. Add 3 to both sides of the equation.
   $$4x + 3 = 2y - 3 + 3$$
   $$4x + 3 = 2y$$

3. Divide each term on both sides of the equation by 2.
   $$\frac{4x}{2} + \frac{3}{2} = \frac{2y}{2}$$

4. Simplify both sides of the equation.
   $$2x + \frac{3}{2} = y$$
   $$y = 2x + \frac{3}{2}$$

To transpose the above formula, we use the same methods as those employed for solving linear equations. The only difference is that in the end we do not obtain a unique (or specific) numerical value for the required variable, but rather an expression in terms of other variables.

Most of the relations that describe real-life situations are non-linear. Consider, for example, any formula for area or volume. A few examples are the area of a circle, $A = \pi r^2$ (non-linear, since it contains $r$ to the power of 2); the area of a triangle, $A = \frac{1}{2}bh$ (non-linear, since it contains the product of two variables); and the volume of a cube, $V = s^3$ (non-linear, since $s$ is cubed).

Non-linear formulas can be transposed by performing identical inverse operations to both sides of the equations.

The inverse of $x^2$ is $\pm\sqrt{x}$, the inverse of $\sqrt{x}$ is $x^2$ and so on.

**WORKED EXAMPLE 9**

Transpose each of the following formulas to make the pronumerals indicated in brackets the subject.

<table>
<thead>
<tr>
<th>a</th>
<th>$A = \frac{4}{3} \pi r^3$</th>
<th>b</th>
<th>$P = \frac{ab - ac}{d}$</th>
<th>c</th>
<th>$m = \sqrt{pq - rs}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>THINK</td>
<td>WRITE</td>
<td>THINK</td>
<td>WRITE</td>
<td>THINK</td>
<td>WRITE</td>
</tr>
<tr>
<td>a 1. Write the equation.</td>
<td>$A = \frac{4}{3} \pi r^2$</td>
<td>a 1. Write the equation.</td>
<td>$A = \frac{4}{3} \pi r^2$</td>
<td>a 1. Write the equation.</td>
<td>$A = \frac{4}{3} \pi r^2$</td>
</tr>
<tr>
<td>2. Multiply both sides of the equation by 3.</td>
<td>$3 \times A = \frac{4}{3} \pi r^2 \times 3$</td>
<td>2. Multiply both sides of the equation by 3.</td>
<td>$3A = \frac{4}{3} \pi r^2$</td>
<td>2. Multiply both sides of the equation by 3.</td>
<td>$3A = \frac{4}{3} \pi r^2$</td>
</tr>
</tbody>
</table>
3. Divide both sides by 4$\pi$.

\[
\frac{3A}{4\pi} = \frac{4\pi r^2}{4\pi} = \frac{3A}{4\pi} = r^2
\]

4. Take the square root of both sides.

Note: From an algebraic point of view we should write $\pm$ in front of the root. However, since $r$ represents a physical quantity (radius of a sphere in this case), it can take only positive values.

b 1. Write the equation.

2. Multiply both sides of the equation by $d$.

3. Take out a common factor of $a$ from the RHS.

4. Divide both sides by $(b - c)$.

5. Write the answer.

Note: Capital $P$ should be used in the answer.

c 1. Write the equation.

2. The inverse of $\sqrt{x}$ is $x^2$, so square both sides.

3. Subtract $pq$ from both sides.

4. Divide both sides by $-r$.

5. Multiply the numerator and denominator by $-1$ (optional).

---

**Exercise 4C Transposition**

1. Transpose each of the following formulas to make the pronumeral indicated in brackets the subject. (Where two pronumerals are indicated, perform a separate transposition for each.)

   a 5$y + 4x = 20$  \hspace{1cm} (x, y)
   b $3x - 4y + 12 = 0$  \hspace{1cm} (y)
   c $m = 3a - 14$  \hspace{1cm} (a)
   d $5p = 2 - 3k$  \hspace{1cm} (k)
   e $\frac{1}{2}a = \frac{3}{4}b$  \hspace{1cm} (a, b)
   f $10 - 3a = 2a - b$  \hspace{1cm} (a, b)
   g $a = 3b - 0.5c$  \hspace{1cm} (c)
   h $\frac{2(a - 3)}{5} = b$  \hspace{1cm} (a)
   i $5(3 - 2d) = 6(f + 4)$  \hspace{1cm} (d, f)
   j $\frac{7(a - 4b)}{3} = \frac{5(b - 2a)}{4}$  \hspace{1cm} (a, b)
   k $\frac{3a}{2} + \frac{2(b + 3a)}{3} = -1$  \hspace{1cm} (a, b)
   l $\frac{2x}{5} - 6 = \frac{3x - 6y}{10}$  \hspace{1cm} (x, y)
2

Transpose each of the following formulas to make the pronumerals indicated in brackets the subject. (Where two pronumerals are indicated, perform a separate transposition for each.)

a \( v^2 = u^2 + as \) \((a, u)\)

b \( S = 4\pi r^2 \) \((r)\)

c \( \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \) \((R, R_1)\)

d \( A = A_0 \left(1 + \frac{r}{100}\right) \) \((r)\)

e \( s = \frac{1}{2}(u + v) \) \((t, u)\)

f \( T = 2\pi \sqrt{\frac{L}{g}} \) \((L, g)\)

g \( C = \sqrt{a^2 + b^2} \) \((b)\)

h \( s = ut + \frac{1}{2}at^2 \) \((a)\)

i \( P = FR \) \((I, R)\)

j \( R = \frac{m_2 - m_1}{t} \) \((m, v_1)\)

Questions 3 to 6 refer to the following information.
A gardener charges a $40 fixed fee for each visit plus $12 per hour of work.

3

Which of the following graphs represents the above information, where \( C \) represents the total cost of a visit and \( t \) the time the gardener worked (in hours)?

4

Which of the following represents the relationship between \( t \) and \( C \)?

A \( C + 40 - 12t = 0 \)

B \( 12t + C = 40 \)

C \( 12t + 40 - C = 0 \)

D \( t = 12C + 40 \)

E \( 40 + 12t + C = 0 \)

5

When the relationship between \( t \) and \( C \) is transposed to make \( t \) the subject, it is then written as:

A \( t = \frac{C + 40}{12} \)

B \( 12t + 40 = C \)

C \( t = \frac{C - 12}{40} \)

D \( t = \frac{C}{40} + \frac{3}{10} \)

E \( \frac{C}{12} - 3\frac{1}{2} = t \)

6

If the total bill came to $79, for how long did the gardener work?

A 3 h

B 3 h 15 min

C 3 h 30 min

D 3 h 45 min

E 4 h

Questions 7 to 10 refer to the following information. The volume of a square-based pyramid with the side of the base \( s \) and the height \( h \) is given by the formula \( V = \frac{1}{3} s^2 h \).

7

The side length of the base of a square-based pyramid with height \( h \) and volume \( V \) is given by:

A \( s = 3\sqrt[3]{\frac{V}{h}} \)

B \( s = \sqrt[3]{\frac{3h}{V}} \)

C \( s = \sqrt[3]{\frac{h}{3V}} \)

D \( s = \sqrt[3]{\frac{V}{3h}} \)

E \( s = \frac{3V}{\sqrt[3]{h}} \)

8

The height of a square-based pyramid with base side length 5 cm and volume 75 cm\(^3\) is:

A 8 cm

B 9 cm

C 10 cm

D 11 cm

E 12 cm

9

If both the side of the base and the height are doubled, the volume is:

A doubled

B tripled

C increased by 4 times

D increased by 6 times

E increased by 8 times
10 MC If the side of the base of a pyramid is doubled but its volume remains unchanged, the height:

A becomes twice as large
B becomes \( \frac{1}{2} \) of the original size
C becomes \( \frac{1}{4} \) of the original size
D becomes 4 times as large
E becomes \( \frac{3}{4} \) of the original size

11 The sum of the interior angles of a regular polygon is given by \( S = (n - 2) \times 180^\circ \), where \( n \) is the number of sides.
   a Transpose the formula to make \( n \) the subject.
   b Use the appropriate formulas to complete the following table:

<table>
<thead>
<tr>
<th>Polygon</th>
<th>Number of sides ( (n) )</th>
<th>Sum of interior angles ( (S) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangle</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>Hexagon</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>Dodecagon</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>Nonagon</td>
<td>9</td>
<td></td>
</tr>
<tr>
<td>Heptagon</td>
<td>7</td>
<td>1080°</td>
</tr>
<tr>
<td></td>
<td></td>
<td>540°</td>
</tr>
<tr>
<td></td>
<td></td>
<td>360°</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1440°</td>
</tr>
</tbody>
</table>

12 The sum of \( n \) terms of an arithmetic sequence is given by the formula \( S = \frac{n}{2} [2a + (n - 1)d] \), where \( a \) is the first number of the sequence and \( d \) is the common difference.
   a Transpose the formula to make \( a \) the subject, and hence find the first term in a sequence that has \( n = 26 \), \( d = 3 \) and \( S = 1079 \).
   b Transpose the formula to make \( d \) the subject, and hence find the common difference of an arithmetic sequence with 20 terms, \( a = 18 \) and \( S = -20 \).

### 4D Solving linear equations and simultaneous linear equations

A linear equation is an equation that contains a pronumeral (unknown value) raised to the power of 1. Such an equation may also be called an equation of the first degree. Examples of linear or first degree equations include:

\[ 2x - 4 = 8, \quad y = 7x - 12 \quad \text{and} \quad y = \frac{x + 5}{3}. \]

Equations of the type

\[ y = \frac{1}{x}, \quad y = \sqrt{x}, \quad 2x^2 - 4 = 8, \quad x^2 + y^2 = 4 \quad \text{and} \quad y = x^3 - 8 \]

are not linear, as they contain pronumerals that are raised to powers other than 1 — in these cases, \( \frac{1}{x} \), \( \sqrt{x} \), 2, 2 and 3, respectively.

A linear equation is an equation that contains a pronumeral raised to the power of 1. It may also be called an equation of the first degree.

### Solving linear equations

When we are asked to solve an equation, we are to find the value of the pronumeral so that when it is substituted into the original equation, it will make the equation a true statement.

Equations are solved by performing a number of inverse operations to both sides of the equation until the value of the unknown is found.
When solving equations, the order of operations process, BODMAS (i.e. Brackets Of Division, Multiplication, Addition, Subtraction), is reversed. We may therefore apply the SAMDOB process (BODMAS in reverse). This means that the operations of subtraction and addition are taken care of first, followed by multiplication and division. Brackets are dealt with last.

**WORKED EXAMPLE 10**

Solve the following equations.

\[ \text{a } 2x - 3 = 4 \quad \text{b } 10 - \frac{3x}{2} = 5 \]

**THINK**

a 1 Write the given equation.  
2 (Optional step.) Rule up a table with two columns to the side of the equation. In the first column, note each of the operations performed on \( x \) in the correct order. In the second column, write the corresponding inverse operation. The arrows indicate which operation to begin with.  
3 Solve the equation by making \( x \) the subject. Add 3 to both sides of the equation.  
4 Divide both sides of the equation by 2.  
5 Simplify.

b 1 Write the given equation.  
2 (Optional step.) As in part a above.  
3 Solve the equation by making \( x \) the subject. Subtract 10 from both sides of the equation.  
4 Multiply both sides of the equation by 2.  
5 Divide both sides of the equation by \(-3\).  
6 Simplify.

**WRITE**

\[ \text{a } 2x - 3 = 4 \]

\[ \begin{array}{|c|c|}
\hline
\text{Operation} & \text{Inverse} \\
\hline
\times 2 & \div 2 \\
\downarrow - 3 & \downarrow + 3 \\
\hline
\end{array} \]

\[ 2x - 3 = 4 \]
\[ 2x - 3 + 3 = 4 + 3 \]
\[ 2x = 7 \]
\[ \frac{2x}{2} = \frac{7}{2} \]
\[ x = 3 \frac{1}{2} \text{ (or 3.5)} \]

\[ \text{b } 10 - \frac{3x}{2} = 5 \]

\[ \begin{array}{|c|c|}
\hline
\text{Operation} & \text{Inverse} \\
\hline
\times 3 & \div 3 \\
+ 2 & - 2 \\
\downarrow + 10 & \downarrow - 10 \\
\hline
\end{array} \]

\[ 10 - \frac{3x}{2} = 5 \]
\[ 10 - 10 - \frac{3x}{2} = 5 - 10 \]
\[ -\frac{3x}{2} = -5 \]
\[ -\frac{3x}{2} \times 2 = -5 \times 2 \]
\[ -3x = -10 \]
\[ \frac{-3x}{-3} = \frac{-10}{-3} \]
\[ x = \frac{10}{3} \]
\[ = 3 \frac{1}{3} \]
Step 2 in Worked example 10 is an optional step that may be used initially to help you become familiar with the process of solving equations.

The answers may be checked by substituting the values obtained back into the original equation or using a calculator.

If the pronumeral appears in the equation more than once, we must collect terms containing the unknown on one side of the equation and all other terms on the other side.

**WORKED EXAMPLE 11**

Solve for \(x\) in the equation \(2x - 4 = 4x + 6\).

**THINK**

1. Write the given equation.  \[2x - 4 = 4x + 6\]
2. Transpose \(4x\) to the LHS of the equation by subtracting it from both sides of the equation.  \[2x - 4x - 4 = 4x - 4x + 6\]
3. Add 4 to both sides of the equation.  \[-2x - 4 + 4 = 6 + 4\]
4. Divide both sides of the equation by \(-2\).  \[\frac{-2x}{-2} = \frac{10}{-2}\]
5. Simplify.  \[x = -5\]

If the equation contains brackets, they should be expanded first. In some cases, however, both sides of the equation can be divided by the coefficient in front of the brackets instead of expanding.

**WORKED EXAMPLE 12**

Solve for \(x\) in \(2(x + 5) = 3(2x - 6)\).

**THINK**

1. Write the given equation.  \[2(x + 5) = 3(2x - 6)\]
2. Expand each of the brackets on both sides of the equation.  \[2x + 10 = 6x - 18\]
3. Transpose \(6x\) to the LHS of the equation by subtracting it from both sides of the equation.  \[2x - 6x + 10 = 6x - 6x - 18\]
4. Subtract 10 from both sides of the equation.  \[-4x + 10 - 10 = -18 - 10\]
5. Divide both sides of the equation by \(-4\).  \[\frac{-4x}{-4} = \frac{-28}{-4}\]
6. Simplify.  \[x = 7\]

If an equation contains a fraction, we should first remove the denominators by multiplying each term of the equation by the lowest common denominator (LCD).

**WORKED EXAMPLE 13**

Find the value of \(x\) that will make the following a true statement: \[\frac{x + 2}{3} = 5 - \frac{x}{2}\].

**THINK**

1. Write the given equation.  \[\frac{x + 2}{3} = 5 - \frac{x}{2}\]
2. Determine the LCD of 2 and 3.  The LCD of 2 and 3 is 6.
3. Multiply each term of the equation by the LCD.
\[ \frac{x + 2}{3} \times 6 = 5 \times 6 - \frac{x}{2} \times 6 \]

4. Simplify both sides of the equation.
\[ \frac{6(x + 2)}{3} = 30 - \frac{6x}{2} \]
\[ 2(x + 2) = 30 - 3x \]

5. Expand the bracket on the LHS of the equation.
\[ 2x + 4 = 30 - 3x \]

6. Add 3x to both sides of the equation.
\[ 2x + 3x + 4 = 30 - 3x + 3x \]
\[ 5x + 4 = 30 \]

7. Subtract 4 from both sides of the equation.
\[ 5x + 4 - 4 = 30 - 4 \]
\[ 5x = 26 \]

8. Divide both sides of the equation by 5.
\[ \frac{5x}{5} = \frac{26}{5} \]

\[ x = 5 \frac{1}{5} \text{ (or 5.2)} \]

Sometimes in equations containing fractions, a pronumeral appears in the denominator. Such equations are solved in the same manner as those in the previous examples. However, care must be taken to identify the value (or values) for which the pronumeral will cause the denominator to be zero. If in the process of obtaining the solution the pronumeral is found to take such a value, it should be discarded.

Even though the process of identifying the value of the pronumeral that causes the denominator to be zero is at this stage merely a precaution, this process should be practised as it will prove useful in future chapters.

WORKED EXAMPLE 14

Solve the following equation for \( x \):
\[ \frac{2}{x} + \frac{3}{2x} = \frac{1}{x - 1}. \]

**THINK**

1. Identify the values of \( x \) that will cause the denominator to be zero.
   *Note:* Once the equation has been solved, values that cause the denominator to be zero will be discarded.

2. Write the given equation.

3. Determine the LCD of \( x \), \( 2x \) and \( x - 1 \).

4. Multiply each term of the equation by the LCD.

5. Simplify both sides of the equation.

6. Expand the bracket on the LHS of the equation.

7. Collect like terms onto the LHS by subtracting \( 2x \) from both sides of the equation.

8. Add 7 to both sides of the equation.
Divide both sides of the equation by 5.

\[
\frac{5x}{5} = \frac{7}{5}
\]

Simplify.

Note: The value of 1.4 is a valid solution.

Simultaneous equations

It is impossible to solve one linear equation with two unknowns. There must be two equations with the same two unknowns for a solution to be found.

Such equations are called simultaneous equations.

Graphical solution of simultaneous equations

If two straight lines intersect, the point of their intersection belongs to both lines, and hence the coordinates of that point \((x, y)\) will represent the solution of two simultaneous equations that define the lines.

When we are solving simultaneous equations graphically, the accuracy of the solution is highly dependent on the quality of the graph. Therefore, all graphs must be drawn on graph paper as accurately as possible.

It is good practice to verify any answer obtained from a graph by substituting it into the original equations or using a CAS calculator.

**WORKED EXAMPLE 15**

Solve the following pairs of simultaneous equations graphically.

a) \(x + 2y = 4\)

b) \(y + 3x = 17\)

\(x - y = 1\)

\(2x - 3y = 4\)

**THINK**

1. Rule up a set of axes. Label the origin and the \(x\)- and \(y\)-axes.

2. Find the \(x\)-intercept for the equation \(x + 2y = 4\) by making \(y = 0\).

\[
\text{x-intercept: when } y = 0, \\
x + 2y = 4 \\
x + 2 \times 0 = 4 \\
x = 4
\]

The \(x\)-intercept is at \((4, 0)\).

3. Find the \(y\)-intercept for the equation \(x + 2y = 4\) by making \(x = 0\). Divide both sides of the equation by 2.

\[
\text{y-intercept: when } x = 0, \\
x + 2y = 4 \\
0 + 2y = 4 \\
2y = 4 \\
y = 2
\]

The \(y\)-intercept is at \((0, 2)\).

(Refer to the graph at step 7.)

4. Plot the points on graph paper and join them with a straight line. Label the graph.

5. Find the \(x\)-intercept for the equation \(x - y = 1\) by making \(y = 0\).

\[
\text{x-intercept: when } y = 0, \\
x - y = 1 \\
x - 0 = 1 \\
x = 1
\]

The \(x\)-intercept is at \((1, 0)\).

6. Find the \(y\)-intercept for the equation \(x - y = 1\) by making \(x = 0\). Multiply both sides of the equation by \(-1\).

\[
\text{y-intercept: when } x = 0, \\
x - y = 1 \\
0 - y = 1 \\
y = -1
\]

The \(y\)-intercept is at \((0, -1)\).
7 Plot the points on graph paper and join them with a straight line. Label the graph.

8 From the graph, read the coordinates of the point of intersection.

9 Verify the answer by substituting the point of intersection into the original equations.

b 1 Find the axial intercepts for $y + 3x = 17$.
Let $x = 0$, then $y = 0$.

2 Find the axial intercepts for $2x - 3y = 4$.
Let $x = 0$, then $y = 0$.

3 Plot the intercepts on graph paper.
Join the axial intercepts for each equation. Label the graph.

4 From the graph, read the coordinates of the point of intersection.

5 Verify your answer.

6 Write the answer.

The point of intersection between the two graphs is (2, 1).

Substitute $x = 2$ and $y = 1$ into $x + 2y = 4$.
LHS $= 2 + 2 	imes 1$ RHS $= 4$
$= 2 + 2$
$= 4$
LHS $= $ RHS
Substitute $x = 2$ and $y = 1$ into $x - y = 1$.
LHS $= 2 - 1$ RHS $= 1$
$= 1$
LHS $= $ RHS
In both cases LHS $= $ RHS; therefore, the solution set (2, 1) is correct.

b $x = 0, y + 0 = 17 \rightarrow y = 17$
y $= 0, 0 + 3x = 17 \rightarrow x = \frac{17}{3}$

$x = 0, 0 - 3y = 4 \rightarrow y = \frac{-4}{3}$
y $= 0, 2x - 0 = 4 \rightarrow x = 2$

Substitute $x = 5$ and $y = 2$ into $y + 3x = 17$.
LHS $= 2 + 3(5)$ RHS $= 17$
$= 2 + 15$
$= 17$
LHS $= $ RHS
Substitute $x = 5$ and $y = 2$ into $2x - 3y = 4$.
LHS $= 2(5) - 3(2)$ RHS $= 4$
$= 10 - 6$
$= 4$
LHS $= $ RHS
In both cases LHS $= $ RHS; therefore, the solution set (5, 2) is correct.

The point of intersection between the two graphs is (5, 2).
Parallel lines
If two equations have the same gradient, they represent parallel lines. Such lines will never meet and so never have a point of intersection (that is, there is no solution).

The pair of equations \( y = 2x + 3 \) and \( y = 2x + 5 \) define two parallel lines; hence, there is no solution. The graph at right demonstrates that the straight lines never intersect.

Coincidental lines
If two lines coincide, then there are an infinite number of solutions. For example, consider the two straight lines given by the equations \( y = 2x + 1 \) and \( 4x - 2y = -2 \).

Rearranging the second equation, we obtain the same line. 

\[
\frac{4x - 2y}{2} = \frac{-2}{2} \quad \frac{-2y}{-2} = \frac{2x}{2} \quad \frac{-2}{-2} = 1
\]

The two equations when graphed represent the same line — they coincide. Therefore, every point on the line will represent the solution, as there is not one unique point that satisfies both equations.

Algebraic solution of simultaneous equations
When using algebra to solve simultaneous equations, the aim is to obtain one equation with one unknown from two equations with two unknowns by various algebraic manipulations. This can be done in two ways — substitution and elimination — as outlined below.

Substitution method
The method of substitution is easy to use when at least one of the equations represents one unknown in terms of the other.

To solve simultaneous equations using the method of substitution:
1. Check that one of the equations is transposed so that one of the unknowns is expressed in terms of the other.
2. Substitute the transposed equation into the second equation.
3. Solve for the unknown variable.

WORKED EXAMPLE 16

Use the method of substitution to solve the simultaneous equations \( y = 2x + 3 \) and \( 4x - y = 5 \).

**THINK**

1. Write the equations one under the other, and number them. 
   \[
y = 2x + 3 \quad [1] \\
4x - y = 5 \quad [2]
\]
2. Substitute the expression for \( y \) (\( 2x + 3 \)) from equation [1] into equation [2].  
   \[
   \text{Substituting (} 2x + 3 \text{)} \text{ into [2]:} \\
   4x - (2x + 3) = 5
   \]
   *Note: By substituting one equation into the other, we are left with one equation and one unknown.*
3. Solve for \( x \). 
   \[
   4x - 2x - 3 = 5 \\
   2x - 3 = 5 \\
   2x = 8 \\
   x = 4
   \]
4. Substitute 4 in place of \( x \) into [1] to find the value of \( y \). 
   \[
   \text{Substituting } x = 4 \text{ into [1]:} \\
   y = 2 \times 4 + 3 \\
   = 8 + 3 \\
   = 11
   \]
5. Evaluate.
6 Answer the question. Solution: \(x = 4, y = 11\), or solution set \((4, 11)\).

7 Verify the answer by substituting the point of intersection into the original equations or using a CAS calculator.

The answer was checked using a CAS calculator and found to be correct.

If neither of the equations give one unknown in terms of the other, we can still use a method of substitution by first transposing one of the equations.

**Elimination method**

As the name suggests, the idea of the elimination method is to eliminate one of the variables. This is done in the following way.

1. Choose the variable you want to eliminate.
2. Make the coefficients of that variable equal in both equations.
3. Eliminate the variable by either adding or subtracting the two equations.

   Once this is done, the resulting equation will contain only one unknown, which then can be easily found.

**WORKED EXAMPLE 17**

Use the elimination method to solve the following simultaneous equations.

\[
\begin{align*}
2x + 3y &= 4 \\
x - 3y &= 2
\end{align*}
\]

**THINK**

1. Write the equations one under the other, and number them.

\[
\begin{align*}
2x + 3y &= 4 \quad [1] \\
x - 3y &= 2 \quad [2]
\end{align*}
\]

2. Add equations [1] and [2] to eliminate \(y\).

   *Note:* \(y\) is eliminated because the coefficients of \(y\) in both equations are equal in magnitude and opposite in sign.

\[
[1] + [2]: \quad 2x + 3y = 4 \\
2 - 3y = 2
\]

3. Divide both sides of the equation by 3.

\[
\frac{3x}{3} = \frac{6}{3} \\
x = 2
\]

4. Substitute the value of \(x\) into equation [2].

   *Note:* \(x = 2\) may be substituted in either equation.

\[
\begin{align*}
2 - 3y &= 2 \\
\not{2} - 3\not{y} &= \not{2} - \not{2} \\
-3y &= 0 \\
\frac{-3y}{-3} &= \frac{0}{-3}
\end{align*}
\]

5. Solve for \(y\).

   (a) Subtract 2 from both sides of the equation.

   (b) Divide both sides of the equation by \(-3\).

\[
\begin{align*}
y &= 0
\end{align*}
\]

6. Answer the question.

   Solution: \(x = 2, y = 0\), or solution set \((2, 0)\).

7. Verify the answer by substituting the point of intersection into the original equations or using a CAS calculator.

   The answer was checked using a CAS calculator and found to be correct.

If there is no pair of equal coefficients, we can make them the same by multiplying or dividing one or both equations by an appropriate number.


**Worked Example 18**

Solve the following simultaneous equations.

\[ 2x + 3y = 4 \]
\[ 3x + 2y = 10 \]

**THINK**

1. Number each equation.

   \[ 2x + 3y = 4 \] [1]
   \[ 3x + 2y = 10 \] [2]

2. To eliminate the variable \( y \), multiply equation [1] by 2 and equation [2] by \(-3\).

   \[ 4x + 6y = 8 \] [1] \times 2
   \[ -9x - 6y = -30 \] [2] \times -3

3. Add the new equations.

   \[ -5x = -22 \]

4. Solve for \( x \).

   \[ x = \frac{22}{5} \]
   \[ = 4 \frac{2}{5} \]

5. To evaluate \( y \), substitute \( x = \frac{22}{5} \) into equation [1] and solve for \( y \).

   \[ 2x + 3y = 4 \] [1]
   \[ 2\left(\frac{22}{5}\right) + 3y = 4 \]
   \[ \frac{44}{5} + 3y = 4 \]
   \[ 3y = \frac{20}{5} - \frac{44}{5} \]
   \[ 3y = \frac{-24}{5} \]
   \[ y = \frac{-8}{5} \]

6. Write the solution.

   \[ x = \frac{22}{5} \text{ and } y = \frac{-8}{5} \text{ or } \]
   \[ x = 4 \frac{2}{5} \text{ and } y = \frac{-13}{5} \]

7. Verify the solution by substituting the values for \( x \) and \( y \) into equation [2].

   \[ 3x + 2y = 10 \] [2]
   \[ 3\left(\frac{22}{5}\right) + 2\left(\frac{-8}{5}\right) = 10 \]
   \[ \frac{66}{5} + \frac{16}{5} = 10 \]
   \[ \frac{82}{5} = 10 \]
   \[ 10 = 10 \]

   Since the LHS = RHS, the solution is verified.

8. Answer the question.

   Solution: \( x = \frac{22}{5}, y = \frac{-8}{5} \text{ or } \left(\frac{22}{5}, \frac{-8}{5}\right) \)

---

**Exercise 4D** Solving linear equations and simultaneous linear equations

1. **WE10** Solve the following equations.

   a. \( x + 3 = -16 \)
   b. \( 3 - 2x = 10 \)
   c. \( -0.2x = 10 \)
   d. \( \frac{-6x}{7} = -5 \)
   e. \( \frac{x - 1}{2} = \) \frac{3}{4} \)
   f. \( \frac{3x}{4} + 11 = 20 \)
   g. \( \frac{-2x}{5} + 3 = -7 \)
   h. \( \frac{x - 9}{7} = 5 \)
   i. \( \frac{6 - x}{13} = 5 \)
   j. \( \frac{11x + 2}{5} = 7 \)
   k. \( 6 - \frac{x}{2} = 15 \)
   l. \( -17 - \frac{2x}{3} = -20 \)
2 WE11 Solve for \( x \).
   a \( 2x = 7 + 9x \)  
   b \( 15x + 22 = -7x \)  
   c \( 12 - 17x = -5x \)  
   d \( 3x + 4 = x - 6 \)  
   e \( -5x + 2 = 3x + 18 \)  
   f \( 5x - 3 = -3 - 5x \)  
   g \( 2x - 7 = -2x + 1 \)  
   h \( 15x + 2 = 13x - 10 \)  
   i \( 8 - 3x = 4 - x \)  
   j \( 13 - 3x = 4 - 6x \)  
   k \( -9 + 11x = -3 - x \)  

3 WE12 Solve for \( x \).
   a \( 4(x - 20) = 16 \)  
   b \( -(5 + 6x) = 13 \)  
   c \( -(2x - 7) = 3x \)  
   d \( 8(2x + 1) = -(7 - 3x) \)  
   e \( 4(x + 3) = 2(7 - 4x) + 5 \)  
   f \( 5(2x - 4) - 3 + 7(2 - x) = 0 \)  

4 WE13 For each of the following, find the value of \( x \) that will make the statement true.
   a \( \frac{x + 4}{3} = \frac{2x + 1}{2} \)  
   b \( \frac{x}{7} - x = 12 \)  
   c \( \frac{x + 2x}{6} - 3 = 0 \)  
   d \( \frac{7x - 1}{4} = \frac{2 - 3x}{5} \)  
   e \( \frac{7x}{3} = \frac{3(2 - 3x)}{2} + \frac{5x}{8} \)  
   f \( \frac{3x - 2}{4} + \frac{7x}{3} - \frac{2x + 1}{2} = \frac{1}{6} \)  
   g \( \frac{7 - x}{4} = \frac{2(3x - 6)}{3} \)  
   h \( \frac{4(6 - 2x)}{9} = 5 - \frac{3(x + 7)}{6} \)  

5 WE14 For each of the following:
   a state which value (or values) of \( x \) will cause the equation to be undefined
   b solve for \( x \).
      i \( \frac{2}{x + 1} - \frac{1}{x} = 4 \)  
      ii \( \frac{3}{x - 1} + \frac{4}{x} = \frac{2}{3x} \)  
      iii \( \frac{2(3x - 6)}{(x - 1)(x + 1)} + \frac{1}{x + 1} = \frac{4}{x - 1} \)  
      iv \( \frac{5}{2x} - \frac{7}{4x} = 1 \)  
      v \( \frac{3(7x - 4)}{x - 2} = -4 \)  
      vi \( \frac{15}{4x} - \frac{11}{8x} = \frac{-3}{x - 3} \)  

6 WE15 Solve the following pairs of simultaneous equations graphically. Verify your answers with a CAS calculator.
   a \( 3x + y = 6, -x - 2y = 2 \)  
   b \( -x + 3y = 3, 2x + 3y = 12 \)  
   c \( x = y + 2, 2y - x = 0 \)  
   d \( 3x + 2y = -6, y = 1 - x \)  

7 MC The pair of simultaneous equations \( 5 - x \) and \( y = x - 1 \) has:
   A 1 solution  
   B 2 solutions  
   C no solutions  
   D an infinite number of solutions  
   E none of these

8 MC The pair of simultaneous equations \( 2x - 2y = 4 \) and \( y - x + 2 = 0 \) has:
   A 1 solution  
   B 2 solutions  
   C no solutions  
   D an infinite number of solutions  
   E none of these

9 Complete the following statements.
   a If two lines with different gradients go through the origin, then the solution to the pair of simultaneous equations defining those lines is \( \text{___________} \) (give coordinates).
   b If two lines have the same gradients but different \( y \)-intercepts, then the pair of simultaneous equations defining such lines will have \( \text{___________} \) solution(s).
   c If two lines are defined by the equation \( y = mx + c \) and have the same value of \( c \) but different values of \( m \), then the solution to the pair of simultaneous equations will be \( \text{___________} \) (give coordinates).

10 WE16 Solve the following pairs of simultaneous equations by the method of substitution.
   a \( y = 2x + 3 \)  
   b \( x = y \)  
   c \( y = 3x - 6 \)  
   d \( 6x + y = 11 \)  
   e \( 6x - 2y = 10 \)  
   f \( y = 16 + 5x \)  
   \( x = 5 - 4y \)  
   \( 2y - 3x = 13 \)  
   \( 3x - 6y = 36 \)  
   \( 2y - 3x = 13 \)  
   \( 7x + 3y = -25 \)
11 Use the method of elimination to solve each of the following.

\[ \begin{align*} 
\text{a} & \quad 2x + y = 3 \\
\text{b} & \quad x - y = 7 \\
\text{c} & \quad 3x - 2y = -1 \\
\text{d} & \quad 4x - y = 9 \\
\text{e} & \quad x + y = -1 \\
\text{f} & \quad 3x - 6y = -9 \\
\text{g} & \quad x + 3y = 7 \\
\text{h} & \quad 2x + 4y = 24 \\
\text{i} & \quad 2x - y = 0 \\
\text{j} & \quad 5x - 3y = -19 \\
\text{k} & \quad -6x + 4y = 8 \\
\text{l} & \quad 2x - 4y = -9 
\end{align*} \]

12 Nathan is solving a pair of simultaneous equations, \( 2x - 3y = 5 \) [1] and \( 3x + 4y = 10 \) [2], using the elimination method. To eliminate one of the variables, he could multiply equation [1] and equation [2] by:

A 2 and 3 respectively  
B 3 and 4 respectively  
C 3 and 2 respectively  
D 5 and 10 respectively  
E 4 and 2 respectively

13 Solve each of the following pairs of equations using the elimination method.

\[ \begin{align*} 
\text{a} & \quad 2x - 3y = 8 \\
\text{b} & \quad x - 2y = -4 \\
\text{c} & \quad \frac{2}{3}x + \frac{1}{3}y = 5 \\
\text{d} & \quad 3x + 4y = 5 \\
\text{e} & \quad 3x + y = 9 \\
\text{f} & \quad 2x - y = -3 \\
\text{g} & \quad 2y - x = -10 \\
\text{h} & \quad 5y - 2x = 4 \\
\text{i} & \quad \frac{1}{3}x + \frac{3}{5}y = 5 \\
\text{j} & \quad x + 3y = -5 \\
\text{k} & \quad 6x + y = 4 \\
\text{l} & \quad 2y + x = 20 
\end{align*} \]

14 Without solving the equation \( \frac{4}{x - 3} + \frac{2}{x + 1} = \frac{1}{x} \), we know that \( x \) will not be equal to:

A 3  
B -1  
C 0  
D 0 or -1  
E -1 or 3 or 0

15 To solve \( \frac{3(x - 1)}{2} - 5 = \frac{4 - 2x}{3} \), each term of the equation could be multiplied by:

A 2  
B 3  
C 4  
D 5  
E 6

16 To solve the equation \( \frac{2x}{3} = 4 \), the operations that must be performed are:

A \( \times \) both sides by 2, then \( + \) by 3  
B \( \times \) both sides by \( \frac{2}{3} \)  
C \( + \) both sides by \( \frac{2}{3} \)  
D \( \times \) both sides by 3, then \( + \) by 4  
E \( \times \) both sides by 4, then \( + \) by 2

17 a The equation of line [1] in the diagram at right is:

A \( y = \frac{2}{3}x - 2 \)  
B \( y = 2x - 2 \)  
C \( y = \frac{1}{2}x - 2 \)  
D \( y = 2x + 1 \)  
E \( y = 2 + 2x \)

b The equation of line [2] in the diagram is:

A \( y = \frac{2}{3}x + 2 \)  
B \( y = \frac{2}{3}x + 3 \)  
C \( y = \frac{-2}{3}x - 2 \)  
D \( 3y = 2x + 6 \)  
E \( 3y + 2x = 6 \)

c The point of intersection of the two lines has the coordinates:

A \( \left( \frac{3}{2}, \frac{1}{2} \right) \)  
B \( \left( 2, \frac{1}{2} \right) \)  
C (2, 1)  
D \( \left( \frac{1}{2}, \frac{1}{2} \right) \)  
E \( \left( 2, \frac{3}{2} \right) \)

18 Find the value of \( z \) such that the solution to the following equation is \( x = 1 \).

\[ \frac{3}{x - 2} - \frac{z}{x + 1} = \frac{8x}{(x - 2)(x + 1)} \]

19 Solve the following equation.

\[ \frac{5}{2x - 2} = \frac{4}{x - 1} + \frac{6}{x - 2} \]

20 Find the values of \( x, y \) and \( z \) in the following three simultaneous equations with three unknowns.

\[ \begin{align*} 
2x + 3y - z &= -7 \\
3x + 2y + z &= 2 \\
x - 4y + 2z &= 15 
\end{align*} \]
4E Applications

Linear equations can often be used to help us in problem solving. This is usually done in the following way.
1. Identify the unknown and choose any convenient pronumeral (usually \( x \)) to represent it.
2. Use the information given in the problem to compose an equation in terms of the pronumeral.
3. Solve the equation to find the value of the pronumeral.
4. Interpret your result by relating the answer back to the problem.

**WORKED EXAMPLE 19**

**If the sum of twice a certain number and 5 is multiplied by 3 and then divided by 7, the result is 9. Find the number.**

**THINK**

1. Assign the pronumeral \( x \) to the unknown value.
   - Let \( x = \) the unknown number.

2. Build the equation according to the information given.
   - (a) Twice the number — this means \( 2 \times \), so write this. \( 2x \)
   - (b) The sum of twice the number and 5 — this means \( 2x + 5 \), so add this on. \( 2x + 5 \)
   - (c) The sum is multiplied by 3 — this means \( 3(2x + 5) \). Add this on. \( 3(2x + 5) \)
     - Note: We include brackets to indicate the order of operations.
   - (d) The result is divided by 7 — this means \( \frac{3(2x + 5)}{7} \). Add this on. \( \frac{3(2x + 5)}{7} \)
   - (e) The result is 9 — which means that all of the previous computations will equal 9. Write this. \( \frac{3(2x + 5)}{7} = 9 \)

3. Solve for \( x \).
   - (a) Multiply both sides of the equation by 7. \( \frac{3(2x + 5)}{7} \times 7 = 9 \times 7 \)
     - \( 3(2x + 5) = 63 \)
   - (b) Divide both sides of the equation by 3 because they are both divisible by 3. \( \frac{3(2x + 5)}{3} = \frac{63}{3} \)
     - \( 2x + 5 = 21 \)
     - \( 2x + 5 - 5 = 21 - 5 \)
     - \( 2x = 16 \)
     - \( 2x = 16 \)
     - \( 2 \)
   - (c) Subtract 5 from both sides of the equation. \( 2x = 16 \)
     - \( 2x = 16 \)
   - (d) Divide both sides of the equation by 2. \( \frac{2x}{2} = \frac{16}{2} \)
     - \( x = 8 \)
   - (e) Simplify.

4. Answer the question.
   - The unknown number is 8.

Sometimes the problem contains more than one unknown. In such cases one of the unknowns is called \( x \) and the other unknowns are then expressed in terms of \( x \).
Sarah is buying tulip bulbs. Red tulip bulbs cost $5.20 each, and yellow tulip bulbs cost $4.70 each. If 22 bulbs cost Sarah $107.40, how many of each type did she buy?

**THINK**

1. **Define the variables.**
   
   *Note:* Since there are 22 bulbs altogether, the number of yellow tulip bulbs is $22 - \text{the number of red tulip bulbs}$; that is, $22 - x$.

2. **Write an expression for the cost of the red tulips.**
   
   *Note:* One red tulip costs $5.20; therefore $x$ red tulips cost $5.20 \times x$.

3. **Write an expression for the cost of the yellow tulips.**
   
   *Note:* One yellow tulip costs $4.70; therefore $22 - x$ tulips cost $4.70 \times (22 - x)$.

4. **Formulate an equation relating the total cost of the red and yellow tulips and the expressions obtained in steps 2 and 3.**

   The total cost of the red and yellow tulip bulbs is $107.40.
   
   Also, the total cost of the red and yellow tulip bulbs is $5.2x + 4.7(22 - x)$.
   
   Therefore, $5.2x + 4.7(22 - x) = 107.4$

**WRITE**

1. **Let** $x = \text{the number of red tulip bulbs}$. 
   
   Let $22 - x = \text{the number of yellow tulip bulbs}$.

2. **Total cost of red tulip bulbs**
   
   $= 5.20 \times x$
   
   $= 5.2x$

3. **Total cost of yellow tulip bulbs**
   
   $= 4.70 \times (22 - x)$
   
   $= 4.7(22 - x)$

4. **Solve the equation.**

   (a) Expand the brackets on the LHS of the equation.
   
   $5.2x + 103.4 - 4.7x = 107.4$

   (b) Collect the like terms on the LHS of the equation.
   
   $0.5x + 103.4 = 107.4$

   (c) Subtract 103.4 from both sides of the equation.
   
   $0.5x + 103.4 - 103.4 = 107.4 - 103.4$
   
   $0.5x = 4$

   (d) Divide both sides of the equation by 0.5.
   
   $\frac{0.5x}{0.5} = \frac{4}{0.5}$
   
   $x = 8$

5. **Interpret the answer obtained.**

   There are 8 red and 14 (that is, $22 - 8$) yellow tulip bulbs.

6. **Answer the question.**

   Sarah purchased 8 red and 14 yellow tulip bulbs.
A train (denoted as train 1) leaves station A and moves in the direction of station B with an average speed of 60 km/h. Half an hour later another train (denoted as train 2) leaves station A and moves in the direction of the first train with an average speed of 70 km/h. Find:

a. the time needed for the second train to catch up with the first train
b. the distance of both trains from station A at that time.

**THINK**

1. Define the variables.
   - Note: Since the first train left half an hour earlier, the time taken for it to reach the meeting point will be \(x + 0.5\).
2. Write the speed of each train.
3. Write the distance travelled by each of the trains from station A to the point of the meeting. (Distance = speed \(\times\) time.)
4. Equate the two expressions for distance.
   - Note: When the second train catches up with the first train, they are the same distance from station A — that is, \(d_1 = d_2\).
5. Write the equation.
6. Solve for \(x\).
7. Substitute 3 in place of \(x\) into either of the two expressions for distance, say into \(d_2\).
8. Evaluate.
9. Answer the questions.

**WRITE**

Let \(x\) be the time taken for train 2 to reach train 1. Therefore, the travelling time, \(t\), for each train is:

Train 1: \(t_1 = x + 0.5\)
Train 2: \(t_2 = x\)

Train 1: \(v_1 = 60\)
Train 2: \(v_2 = 70\)

Train 1: \(d_1 = 60(x + 0.5)\)
Train 2: \(d_2 = 70x\)

When the second train catches up with the first train, \(d_1 = d_2\).

\[
60(x + 0.5) = 70x \\
60x + 30 = 70x \\
10x = 30 \\
x = 3
\]

Substitute \(x = 3\) into \(d_2 = 70x\)

\[
d_2 = 70 \times 3 = 210
\]

a. The second train will catch up with the first train 3 hours after leaving station A.

b. Both trains will be 210 km from station A.

Simultaneous equations are used to solve a variety of problems containing more than one unknown. Here is a simple algorithm that can be applied to any of them:

1. Identify the variables.
2. Set up simultaneous equations by transforming written information into algebraic sentences.
3. Solve the equations by using the substitution, elimination or graphical methods.
4. Interpret your answer by referring back to the original problem.

**WORKED EXAMPLE 22**

Find two consecutive numbers that add up to 99.

**THINK**

1. Define the two variables.
2. Formulate two equations from the information given and number them.
   - Note: Consecutive numbers follow one another and differ by 1. Hence, if \(x\) is the first number, the next number will be \(x + 1\) — that is, \(y = x + 1\).

**WRITE**

Let \(x\) be the first number.
Let \(y\) be the second number.

\[
x + y = 99 \quad [1] \\
y = x + 1 \quad [2]
\]
Substitute the expression \((x + 1)\) from equation [2] for \(y\) into equation [1].

Substituting \((x + 1)\) into [1]:
\[x + x + 1 = 99\]

Solve for \(x\).

(a) Simplify the LHS of the equation by collecting like terms.
\[2x + 1 = 99\]

(b) Subtract 1 from both sides of the equation.
\[2x + 1 - 1 = 99 - 1\]
\[2x = 98\]

(c) Divide both sides of the equation by 2.
\[\frac{2x}{2} = \frac{98}{2}\]
\[x = 49\]

Substitute 49 in place of \(x\) into equation [1] to find the value of \(y\).

Substituting \(x = 49\) into equation [2]:
\[y = x + 1\]
\[y = 49 + 1\]
\[y = 50\]

Verify the answer by checking that the two values are consecutive and that they sum 99.

49 and 50 are consecutive numbers.
\[49 + 50 = 99\]
The obtained values satisfy the problem.

Answer the question.
The two consecutive numbers that add up to 99 are 49 and 50.

**WORKED EXAMPLE 23**

**Two hamburgers and a packet of chips cost $8.20, and one hamburger and two packets of chips cost $5.90. Find the cost of a packet of chips and a hamburger.**

**THINK**

1. Define the two variables.

2. Formulate an equation from the first sentence and call it [1].

   \[2x + y = 8.20\] \[\text{[1]}\]

   *Note: One hamburger costs $x, two hamburgers cost $2x. Thus, the total cost of two hamburgers and one packet of chips is } 2x + y \text{ and it is equal to $8.20.}*

3. Formulate an equation from the second sentence and call it [2].

   \[x + 2y = 5.90\] \[\text{[2]}\]

   *Note: One packet of chips costs $y, two packets cost $2y. Thus, the total cost of two packets of chips and one hamburger is } x + 2y \text{ and it is equal to $5.90.}*

4. To eliminate the variable \(x\), multiply equation [2] by \(-2\).

   \[-2x - 4y = -11.80\] \[\text{[2] \times -2}\]

5. Add the new equations.

   \[\begin{align*}
   2x + y &= 8.20 \quad \text{[1]} \\
   -2x - 4y &= -11.80 \quad \text{[2] \times -2} \\
   \hline
   -3y &= -3.60
   \end{align*}\]

6. Solve for \(y\).

   \[y = 1.20\]
To evaluate \( x \), substitute \( y = 1.20 \) into equation [1] and solve for \( x \).

\[
2x + y = 8.20 \quad [1]
\]

\[
2x + 1.20 = 8.20
\]

\[
2x = 7.00
\]

\[
x = 3.50
\]

Answer the question and include appropriate units.

A hamburger costs $3.50 and a packet of chips costs $1.20.

It is extremely important to be consistent with the use of units while setting up equations. For example, if the cost of each item is expressed in cents, then the total cost must also be expressed in cents.

**Exercise 4E Applications**

1. **WE19** The average of three consecutive odd numbers is -3. Find the largest number.

2. Half of a certain number is subtracted from 26 and the result is then tripled, and the answer is 18. Find the number.

3. The sum of one-third of a number and 5 is 27. Find the number.

4. **WE20** Fiona is buying tulip bulbs. Red tulip bulbs cost $6.40 each and yellow tulip bulbs cost $5.20 each. If 28 bulbs cost Fiona $167.20, how many of each type did she buy?

5. A rectangle is 2.5 times as long as it is wide. Find the dimensions of the rectangle if its perimeter is 56 cm.

6. In an isosceles triangle, two sides of equal length are together 8 cm longer than the third side. If the perimeter of the triangle is 32 cm, what is the length of each side?

7. In a scalene triangle, the first angle is 3 times as large as the second, and the third angle is 20° smaller than the second. Find the size of each angle; hence, name the triangle according to its angles’ sizes.

8. All items at a clothing store have been reduced by 15%. If Stephanie purchased a shirt at the reduced price of $84.15, what was its original price?

9. **MC**
   
   a. If 7 times a number subtracted from 52 gives 3, then the number is:
   
   \[
   \begin{array}{cccccc}
   \text{A} & -7 & \text{B} & 7 & \text{C} & 8 & \text{D} & 6 & \text{E} & 7 \frac{6}{7} \\
   \end{array}
   \]
   
   b. The sum of one-quarter of a number and 10 is 15. The value of the number is:
   
   \[
   \begin{array}{cccccc}
   \text{A} & 100 & \text{B} & 50 & \text{C} & 40 & \text{D} & 20 & \text{E} & 10 \\
   \end{array}
   \]

10. **a** I am 3 times as old as my cousin Carla, who is \( \frac{3}{7} \) times as old as my daughter Nina. If our total ages are 43 years, how old is my cousin?

   **b** Another cousin, Zara, is Carla’s older sister. Zara is as many times as old as my daughter Nina as the number of years that she is older than Carla. How old is my other cousin?

11. Simon is only 16 years old, but he has already lived in four different countries because of his father’s job. He was born and spent a few years of his early childhood in the USA, then the family moved to Germany, where he stayed one year longer than he had in the USA. After that, he lived in London for twice as long as he had in Germany. Finally, they came to live in Melbourne. So far, he has been in Australia for 2 years less than he lived in America.

   **a** At what age did Simon leave his country of birth?

   **b** For how long did Simon live in each country?
12 In the storeroom of a fruit shop there were two boxes of apples, one of Golden Delicious and the other of Jonathans, which were to be sold at $2.80/kg and $3.50/kg respectively. The apples, however, were accidentally mixed together and, instead of sorting them out, the owner decided to sell them as they were. So as not to make a loss, he sold the mixed apples at $3.10/kg. How many kilograms of each type of apple were there if together they weighed 35 kg?

13 Alex and Nat are going for a bike ride. Nat can ride at 10 km/h, and Alex can develop a maximum speed of 12 km/h if he needs to. Nat leaves home at 10 am, while Alex stays behind for 15 minutes and then sets out to catch up with Nat. When will Alex reach Nat, assuming that both of them are riding at their maximum speed?

14 Samuel is paddling with a constant speed towards a certain place he has marked on his map. With the aid of a current (which has a speed of 2 km/h) it takes him only 1 h 20 min to reach his destination. However, on the way back he has to paddle against the current, and it then takes him 4 h to reach his starting point. Find Samuel’s speed on the still water.

15 One administrative assistant can type 1.5 times as fast as another. If they both work together, they can finish a certain job in 6 hours. However, the slower one working alone will need 15 hours to finish the same job. How many hours will the quicker assistant alone need to complete the job?

16 Maya needs to renovate her house. She has enough money to pay a plumber for 28 days or a carpenter for 21 days. For how many days can she pay the renovators if they both work at the same time? If Maya’s next pay cheque will come in 2 weeks, can she afford to hire both specialists until then?

17 In a particular school, a number of VCE students obtained a tertiary entrance score higher than 99.4, and 15% more students obtained a score higher than 99.0 but lower than 99.4. If there were 43 students whose tertiary entrance scores were above 99.0, how many of those obtained a score above 99.4?

18 Find two consecutive numbers that add up to 89.

19 When three times the first number is added to twice the second number, the result is 13. Four times the difference of those numbers is 44. Find the numbers.

20 Half of the sum of two numbers is 6 less than the first number. One-third of their difference is one less than the second number. Find the numbers.

21 Five times the first number is twice as large as four times the second number. When the difference of the two numbers is multiplied by 20, the result is 3. Find the numbers.

22 A rectangle’s length is 2 cm more than its width. If the perimeter of a rectangle is 24 cm, find its dimensions and, hence, its area.

23 In the rectangle at right, find the values of $x$ and $y$. Hence, determine the perimeter.

24 The sides of an equilateral triangle have the following lengths: $(x + y)$ cm, $(2x - 3)$ cm and $(3y - 1)$ cm. Find the perimeter of the triangle.

25 The perimeter of a rhombus $ABCD$ is 10 cm longer than the perimeter of an isosceles triangle $ABC$. Find the length of $AC$, the diagonal of a rhombus, if it is 2 cm smaller than its side.

26 A table consists of 2 columns and 5 rows. Each of its cells is a rectangle with length $x$ cm and width $y$ cm. The perimeter of the table is 70 cm and the total length of interior lines is 65 cm.
  a Draw a diagram to represent the above information.
  b Find the dimensions of each cell and comment on its shape.
27 Phuong conducts a survey. She asks 72 people whether or not they use the internet at home. There were three times as many people who answered ‘Yes’ as those who answered ‘No’. Find the number of people in each category and hence help Phuong to complete the following statement: ‘According to the survey, _____________ (insert fraction) of the population uses the internet at home.’

28 At the end of the day, two shop assistants compare their sales. One sold 5 toasters and 2 sandwich-makers for a total of $149.65, while the other sold 3 of each for a total value of $134.70. Find the price of each item.

29 In an aquatic centre, pool and spa entry is $3.50, and pool, spa, sauna and steam room entry is $5.20. At the end of the day, a cashier finds that she sold 193 tickets altogether, and her takings are 40c short of $800. How many of each type of ticket were sold?

30 Spiro empties his piggy bank. He has 42 coins, some of which are 5c coins and some of which are 10c coins, to the total value of $2.50. How many 5c coins and how many 10c coins does he have?

31 Maya and Rose are buying meat for a picnic. Maya’s family likes lamb more than pork, so she buys 3 kg of lamb and only half as much pork. Rose’s family have different tastes, so she buys 4.5 kg of pork and one-third as much lamb. If Maya spends $13.50, which is $8.25 less than Rose spends, what is the cost of 1 kg of each type of meat?

32 Bella and Boris are celebrating their 25th wedding anniversary. Today, their combined age is exactly 100. If Boris is 4 years older than Bella, how old was his bride on the day of their wedding?

33 Sasha is making dim sims and spring rolls for his guests. He is going to prepare everything first and then cook. On average it takes 0.2 hours to prepare one portion of dim sims and 0.25 hours to prepare one portion of spring rolls. He needs 0.05 hours and 0.15 hours to cook each portion of dim sims and spring rolls respectively. If he spends 2 hours on preparation and 51 minutes on cooking, how many portions of dim sims and spring rolls does Sasha make?

### 4F Algebraic fractions

Algebraic fractions are fractions that contain a pronumerals. Performing operations involving these fractions involves the same rules as those for numerical fractions.

#### Addition and subtraction

Fractions can only be added or subtracted when they have a common denominator.

**WORKED EXAMPLE 24**

Simplify:

\[
a. \quad \frac{x}{2} + \frac{x}{5}
\]

**THINK**

\[
a. \quad \text{Find the lowest common denominator.}
\]

\[
2 \quad \text{Multiply the first term by } \frac{5}{5} \text{ and the second term by } \frac{2}{2}.
\]

\[
3 \quad \text{Add the numerators.}
\]

**WRITE**

\[
a. \quad \frac{2 \times 5}{5 \times 2} = \frac{10}{10}
\]

\[
\text{Lowest common denominator } = 10
\]

\[
\frac{5 \times \frac{x}{2}}{10} + \frac{2 \times \frac{x}{5}}{10}
\]

\[
= \frac{5x}{10} + \frac{2x}{10}
\]

\[
= \frac{7x}{10}
\]

\[
b. \quad \frac{x^2}{2x} + \frac{6}{2x}
\]

**THINK**

\[
b. \quad \text{Find the lowest common denominator.}
\]

\[
2 \quad \text{Multiply the first term by } \frac{x}{x} \text{ and the second term by } \frac{2}{2}.
\]

\[
3 \quad \text{Add the numerators.}
\]

\[
b. \quad \text{Lowest common denominator } = 2x
\]

\[
\frac{x^2 \times 2}{2x} + \frac{6 \times x}{2x}
\]

\[
= \frac{x^2 + 6}{2x}
\]
WORKED EXAMPLE 25

Simplify:

a \( \frac{3}{x} - \frac{2}{x-1} \)  

b \( \frac{2a}{x+3} + \frac{2}{x-3} \).

**THINK**

a 1. Find the lowest common denominator.

2. Multiply the first term by \( \frac{x-1}{x-1} \) and the second term by \( \frac{x}{x} \).

3. Add the numerators.

4. Expand and simplify the numerator.

b 1. Find the lowest common denominator.

2. Multiply the first term by \( \frac{x-3}{x-3} \) and the second term by \( \frac{x+3}{x+3} \).

3. Add the numerators.

4. Expand the numerator.

**WRITE**

a 1. Lowest common denominator = \( x(x-1) \)

\[
\frac{3(x-1)}{x(x-1)} - \frac{2x}{x(x-1)} = \frac{3(x-1) - 2x}{x(x-1)} = \frac{x-3}{x(x-1)}
\]

b 1. Lowest common denominator = \( (x+3)(x-3) \), or \( x^2 - 9 \)

\[
\frac{2a(x-3)}{x^2-9} + \frac{2(x+3)}{x^2-9} = \frac{2a(x-3) + 2(x+3)}{x^2-9} = \frac{2ax - 6a + 2x + 6}{x^2-9}
\]

**Multiplication and division**

When multiplying, cancelling vertically or diagonally helps to simplify expressions before multiplying top and bottom. This may involve factorising expressions to identify common factors.

WORKED EXAMPLE 26

Simplify:

a \( \frac{3x^2}{4} \times \frac{20}{9x} \)  

b \( \frac{x^2+4}{6y^2} \times \frac{2y}{5x^2 + 20} \).

**THINK**

a 1. Cancel common factors between numerators and denominators, then simplify.

2. Multiply numerators together and denominators together.

b 1. Factorise the denominator.

2. Cancel common factors and simplify.

3. Multiply numerators together and denominators together.

**WRITE**

a 1. Common factors: 3, \( x \), 4

\[
\frac{3x^2}{4} \times \frac{20}{9x} = \frac{x}{9} \times \frac{5}{3} = \frac{5x}{27}
\]

b 1. Factorise the denominator.

\[
\frac{x^2+4}{6y^2} \times \frac{2y}{5(x^2+4)} = \frac{1}{3y} \times \frac{1}{5} = \frac{1}{15y}
\]
Exercise 4F  Algebraic fractions

1 WE24 Simplify the following expressions.

\[ \begin{align*}
\text{a} & \quad \frac{3a}{7} + \frac{2a}{4} \\
\text{b} & \quad \frac{5b}{3} + \frac{5}{4} \\
\text{c} & \quad \frac{4d}{5} - \frac{4}{3} \\
\text{d} & \quad \frac{3g}{2} - \frac{4g}{3} \\
\text{e} & \quad \frac{4}{5} + \frac{2k}{3} + \frac{5}{2} \\
\text{f} & \quad \frac{2k}{3} + \frac{5}{2} \\
\text{g} & \quad \frac{2}{7} - \frac{m}{n} \\
\text{h} & \quad \frac{1}{3n} - \frac{2n}{3}
\end{align*} \]

2 WE25 Simplify the following expressions.

\[ \begin{align*}
\text{a} & \quad \frac{2}{p} + \frac{3}{p+2} \\
\text{b} & \quad \frac{3}{2q} + \frac{5}{q+5} \\
\text{c} & \quad \frac{4}{(r+1)} + \frac{3}{(r-2)} \\
\text{d} & \quad \frac{1}{(s-3)} - \frac{7}{(s+4)} \\
\text{e} & \quad \frac{3}{(2t+3)} - \frac{2}{(t-1)} \\
\text{f} & \quad \frac{3}{(2v-3)} + \frac{5v}{8} \\
\text{g} & \quad \frac{3w}{2} - \frac{5}{(w-2)} \\
\text{h} & \quad \frac{5(x-1)}{3} + \frac{(x+3)}{2}
\end{align*} \]

3 WE25 Simplify the following expressions.

\[ \begin{align*}
\text{a} & \quad \frac{3}{(y+3)} - \frac{7}{(y-3)} \\
\text{b} & \quad \frac{1}{(z+2)} + \frac{5}{(z-2)} \\
\text{c} & \quad \frac{1}{(3-2x)} + \frac{4}{(x-2)} \\
\text{d} & \quad \frac{3}{(1-y)} + \frac{2}{(y+3)} \\
\text{e} & \quad \frac{2}{(a+3)^2} + \frac{5}{(a+3)} \\
\text{f} & \quad \frac{2}{(3b-2)} - \frac{7}{(3b-2)^2}
\end{align*} \]

4 MC When simplifying the expression \( \frac{2}{(x-3)^2} - \frac{2}{3(x-3)} \), the lowest common denominator is:

A \( x-3 \)  
B \( 3(x-3) \)  
C \( 3(x-3)^2 \)

5 MC Simplifying \( \frac{2a^2-a}{2a^3b^3} + \frac{4a^2-4a+1}{(2a^2b)^2} \) gives:

A \( \frac{2a^2}{b(2a-1)} \)  
B \( \frac{4a^2}{b(2a-1)} \)  
C \( \frac{2a^2b}{(2a-1)} \)

6 MC Simplifying \( \frac{9-e^2}{e^3} + \frac{2(e+3)}{e^3} \) gives:

A \( \frac{3-e}{2} \)  
B \( \frac{3-e}{2e} \)  
C \( \frac{e(3-e)}{2} \)  
D \( \frac{3+e}{2} \)  
E \( \frac{3+e}{2e} \)

7 WE26 Simplify the following expressions.

\[ \begin{align*}
\text{a} & \quad \frac{2x^3}{x+2} \times \frac{2(x+2)}{10x^2} \\
\text{b} & \quad \frac{3b}{3(b+5)} \times \frac{2(b+5)}{8b^2} \\
\text{c} & \quad \frac{d^2+5}{3d^3} \times \frac{12}{2d^2+10} \\
\text{d} & \quad \frac{-2a^2}{b(2a-1)} \\
\text{e} & \quad \frac{g^2(2-g)}{6} \times \frac{3g}{4-g^2} \\
\text{f} & \quad \frac{7h(h+2)}{2} \times \frac{12h^2}{7h^3+14h^2}
\end{align*} \]

8 MC Simplify the following expressions.

\[ \begin{align*}
\text{a} & \quad \frac{(j-3)(j+2)}{3(j+7)^2} \times \frac{12(j-3)(j+7)}{2(j+2)^2} \\
\text{b} & \quad \frac{2(k+1)^2(k-2)}{5(k+5)} \times \frac{15(k+5)(k-2)}{3(k+1)^3} \\
\text{c} & \quad \frac{2m^2-m-3}{6(m-1)} \times \frac{3m}{2m(2m-3)^2} \\
\text{d} & \quad \frac{-(n+1)^2}{6n^2} \times \frac{9n}{n^2-1} \\
\text{e} & \quad \frac{94}{(q-2)(q+3)} + \frac{(q-2)^2}{3q+15} \\
\text{f} & \quad \frac{3s^2-4}{18s-27} \times \frac{2s-4}{-(4s^2-9)}
\end{align*} \]
4G Linear literal equations

Literal equations are those that are written in terms of pronumerals such that no unique numerical solution will be possible, but rather an expression containing these pronumerals. An equation such as \( mx - n = p \) could be described as a linear literal equation in \( x \), as it is linear and contains pronumerals rather than numbers. (Note, that in this case \( x \) is defined as the variable.)

A solution to a literal equation can be determined algebraically by the use of inverse operations just as for a numerical equation. The difference is that the solution will be a general one — that is, in terms of the pronumerals.

In the example above, the solution to this equation can be obtained by isolating \( x \) as the subject as follows:

\[
mx - n = p \\
mx = p + n \\
x = \frac{p + n}{m}
\]

Note that literal equations always contain at least one pronumeral (apart from the variable), but they may also contain numerals.

**WORKED EXAMPLE 27**

Solve for \( x \).

\[
a \quad \frac{ax}{b} - c = d \\
b \quad \frac{m}{x-a} = \frac{3n}{x}
\]

**THINK**

\[
ar 1 \text{ Add } c \text{ to both sides.} \\
r 2 \text{ Multiply both sides by } b. \\
r 3 \text{ Divide both sides by } a.
\]

\[
b 1 \text{ Multiply both sides by } x-a. \\
b 2 \text{ Multiply both sides by } x. \\
b 3 \text{ Expand.} \\
b 4 \text{ Collect } x \text{ terms.} \\
b 5 \text{ Factorise.} \\
b 3 \text{ Divide both sides by } 3n-m.
\]

**WRITE**

\[
a \quad \frac{ax}{b} - c = d \\
\quad \frac{ax}{b} = d + c \\
\quad ax = b(d + c) \\
\quad x = \frac{b(d + c)}{a}
\]

\[
b \quad \frac{m}{x-a} = \frac{3n}{x} \\
\quad m = \frac{3n(x-a)}{x}
\]

\[
mx = 3n(x-a) \\
3nx - mx = 3na \\
x(3n-m) = 3na \\
x = \frac{3na}{3n-m}
\]

Solving simultaneous literal equations requires the same method as numerical linear equations, namely, substitution or elimination methods. The solutions will be in terms of the pronumerals.
Solve for $x$ and $y$.

$$ax - by = -4$$
$$2ax - 3by = 6$$

**THINK**

1. Assign a number to each equation.

   $$ax - by = -4 \quad [1]$$
   $$2ax - 3by = 6 \quad [2]$$


   $$2ax - 2by = -8 \quad [3]$$


   $$-by = -14 \quad [4]$$

4. Solve for $y$.

   $$y = \frac{-14}{b}$$

5. Substitute this value of $y$ into equation [1] and solve for $x$.

   $$ax + 14 = -4$$
   $$ax = -18$$
   $$x = \frac{-18}{a}$$

6. State the solution.

   $$x = \frac{-18}{a}, \ y = \frac{-14}{b}$$

---

**Exercise 4G  Linear literal equations**

1. **WE27** Solve for $x$.

   a) \( \frac{x}{b} = c \)
   
   b) \( \frac{2x}{w} = y \)
   
   c) \( \frac{2(x-m)}{n} = p \)

   d) \( \frac{x+r}{s} = 3t \)
   
   e) \( \frac{d}{x} - f = g \)
   
   f) \( \frac{3k}{x+l} + l = 4 \)

   g) \( ax + b = \frac{1}{c} \)
   
   h) \( 2bx - c = 4a \)
   
   i) \( a(b-x) = b - a \)

   j) \( \frac{1}{x} - \frac{m}{n} = m \)
   
   k) \( r(x-s) = \frac{1}{b} \)
   
   l) \( nx - p(x-q) = n(x+p) \)

   m) \( \frac{1}{x+d} + e = \frac{1}{f(x+d)} \)
   
   n) \( \frac{b x + c x}{n + m} = n \)
   
   o) \( \frac{c + x}{x} + \frac{d}{2x} = \frac{e}{3x} \)

   p) \( \frac{x}{dx} - \frac{e}{e} = x - f \)

2. **MC** The solution to \( \frac{x}{a} - \frac{y}{b} = c \) in terms of $x$ is:

   A) \( \frac{abc - ya}{b} \)
   
   B) \( \frac{a(c-a)}{b} \)
   
   C) \( \frac{ac(c-ay)}{b} \)

   D) \( \frac{a(cb+y)}{b} \)
   
   E) \( \frac{ac(c+ay)}{b} \)

3. **WE28** Solve the following simultaneous equations.

   a) \( ax + by = a^2 + b^2 \)
   
   b) \( ax + by = a^2 - ab + 2b^2 \)

   c) \( \frac{x}{a} + \frac{y}{b} = 1 \)
   
   d) \( \frac{x+y}{a} = \frac{a+b}{a} + 1 \)

   c) \( \frac{x}{a} - \frac{y}{b} = 3 \)
   
   d) \( \frac{x+y}{2a} = \frac{a+b}{2a} \)
The sum of $n$ terms of an arithmetic sequence is given by the formula $S = \frac{n}{2}[2a + (n-1)d]$, where $a$ is the first number of the sequence and $d$ is the common difference.

a Transpose the formula to make $a$ the subject, and hence find the first term in a sequence that has $n = 26$, $d = 3$ and $S = 1079$.

b Transpose the formula to make $d$ the subject, and hence find the common difference of an arithmetic sequence with 20 terms, $a = 18$ and $S = -20$. 

\[
e \frac{x}{a} + by = \frac{a + b^2}{a} \\
\frac{x}{b} + y = \frac{a^2 + b^2}{ab} \\
a^2x + by = ab - 2b^2 \\
bx + \frac{y}{a} = \frac{b^2 - 2b}{a} \\
\frac{x}{a} - by = \frac{-b}{a} \\
\frac{x}{b} + ay = \frac{a - b}{b} \\
\]

\[(a - b)x + \frac{y}{b} = 4 \\
(b - a)x + \frac{y}{3b} = 0 \\
(\frac{a}{b})x + by = \frac{-b}{a} \\
\frac{x}{b} + ay = \frac{a - b}{b} \]
Review of index laws

\[ a^0 = 1 \]
\[ a^1 = a \]
\[ a^{-m} = \frac{1}{a^m} \]
\[ a^m \times a^n = a^{m+n} \]
\[ a^m + a^n = \frac{a^m}{a^n} = a^{m-n} \]
\[ (a^m)^n = a^{m \times n} \]
\[ \frac{m}{a^n} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m \]
\[ (a \times b)^n = a^n \times b^n \]

Standard form and significant figures

- The number of significant figures in a number can be determined by considering each of the following rules:
  1. Significant figures are counted from the first non-zero digit (1–9).
  2. Any zeros at the end of the number after the decimal point are considered to be significant.
  3. The trailing zeros at the end of a number are not considered significant.
  4. All zeros between two non-zero digits are always significant.
- When performing calculations associated with significant figures, the following rules apply:
  1. When adding or subtracting numbers, count the number of decimal places to determine the number of significant figures. The answer cannot contain more places after the decimal point than the least number of decimal places in the numbers being added or subtracted.
  2. When multiplying or dividing numbers, count the number of significant figures. The answer cannot contain more significant figures than the number being multiplied or divided with the least number of significant figures.

Transposition

- Transposition is the rearrangement of terms within a formula.
- The subject of the formula is the variable that is by itself on one side of the equation, while all other variables are on the other side.

Solving linear equations and simultaneous linear equations

- Linear equations can be solved by using inverse operations. When solving linear equations, the order of operations process, BODMAS, is reversed.
- Simultaneous linear equations can be solved either graphically or algebraically.
  1. Graphical method:
     Draw the straight lines representing the equations and find the coordinates of the point of intersection.
  2. Algebraic methods:
     (a) Substitution: Transpose one of the equations so that one of the unknowns is expressed in terms of the other and substitute into the second equation.
     (b) Elimination: Equate the coefficients of one unknown and eliminate it by either adding or subtracting the two equations.

Applications using linear equations

- To solve worded problems using linear equations, follow these steps:
  1. Identify the variables.
  2. Set up an equation by transforming the written information into an algebraic statement or statements.
  3. Solve the equation.
  4. Interpret the result by relating the answer back to the original problem.
### Applications using simultaneous equations

1. Identify and define the variables.
2. Transform written information into algebraic statements.
3. Solve the pair of equations graphically or algebraically using the methods of substitution or elimination.
4. Interpret the result by relating the answer back to the problem.
5. Always make sure the numbers in the equations are in the same units.

### Algebraic fractions

- To add or subtract algebraic fractions, first find a common denominator and solve by adding or subtracting the numerator.
- If the pronumeral is in the denominator, you can generally find the common denominator by multiplying the denominators together.
- When multiplying or dividing, cancel down before multiplying through numerators and denominators.

### Linear literal equations

- Linear literal equations are equations that have a variable, such as $x$, with constants and coefficients that are either numbers or pronumerals.
- To solve linear literal equations, use the inverse operations to obtain an equation with the variable as the subject in terms of the other pronumerals.
- The solution will be in terms of the pronumerals.
- Simultaneous linear literal equations can be solved using elimination or substitution methods. The solutions will be in terms of the pronumerals.
1. Simplify \( \frac{2}{x^3} \div \frac{4}{x^3} \).  

2. Write the following in standard form and simplify. 
   \( a \) \( \frac{450000}{120000000} \) \( b \) \( 0.000012 \times 0.34 \) \( c \) \( 245 \times 17000 \).  

3. Calculate the following to the correct number of significant figures. 
   \( a \) \( 3.2418 + 103.27 \) \( b \) \( 1.0065 \times 1200 \).  

4. Transpose each of the following formulas to make the pronumeral indicated in brackets the subject. (If two pronumerals are indicated, make a separate transposition for each.)  
   \( a \) \( 6x - 12y + 15 = 0 \) \( x \) \( b \) \( 7(3 - 4d) = \frac{8(e + 7)}{3} - 3 \) \( d, e \)  

5. Solve for \( x \). \( \frac{2x - 3}{5} - \frac{6 - 3x}{4} = -2 + \frac{7x}{3} \).  

6. a. Find the equations of the two lines shown on the diagram.  
   b. Find the coordinates of the point of intersection (the diagram is not drawn to scale).  

7. Solve the following simultaneous equations.  
   \( a \) \( 6x + 2y = 12 \) \( x - 2y = 2 \) \( b \) \( 8y - 24 = 4x \) \( 7x + 3y = -25 \) \( c \) \( 15 - 3x - 3y = 30 \) \( 2x + y = -4 \).  

8. Before opening the store, a cashier makes sure that his register contains at least $5 in change. He counts a number of 10c coins, twice as many 5c coins and 4 times as many 20c coins, to the total value of exactly $5. How many coins of each type does he count?  

9. A building company charges a $2300 set fee plus $500 a day while it is working on a project within the time limits that are specified by a contract. If the project is completed earlier than the set time, the company will still charge $500 for each of the remaining days. However, if the project is not completed by the due date, the company will pay a $135 penalty for each extra day until the work is done. From the given information, construct a set of formulas for the total cost of work, \( T \); the number of days it takes to complete the job according to the contract, \( n \); and the number of extra days, \( e \).  

10. Jessica is 3 years older than Rebecca. In 5 years she will be 3 times as old as Rebecca was 2 years ago. Find the girls’ present ages.  

11. Simplify the following.  
   \( a \) \( \frac{5m - 4}{2} - \frac{m + 6}{3} \) \( b \) \( \frac{1}{3x - 4} - \frac{4}{x} \) \( c \) \( \frac{3}{(x + 1)(x + 2)} + \frac{2}{(x - 3)(x + 1)} \).  

12. Simplify \( \frac{3(g - 2)(g + 3)^2}{4(g - 1)^4} \times \frac{12(g - 1)^2(g + 3)^3}{(g - 2)}. \)  

13. Simplify \( \frac{25w - 15}{2w - 8} + \frac{w^2 - 9}{4(w^2 - 16)}. \)  

14. Solve for \( x \).  
   \( a \) \( \frac{b - ax}{g} = mn \) \( b \) \( \frac{px - w}{3} + \frac{x}{k} = pk \).  

15. Solve the following simultaneous equations for \( x \) and \( y \).  
   \( a \) \( 2ax - by = 1 + 2b^2 \) \( b \) \( bx - y = \frac{5b}{a} \).
1 \( \left( \frac{3m^2}{n^4} \right)^3 \) is equal to:

A \( \frac{9m^5}{n^7} \)  
B \( \frac{27m^6}{n^{12}} \)  
C \( \frac{3m^3}{n^3} \)  
D \( \frac{6m^5}{n^7} \)  
E \( \frac{9m^6}{n^{12}} \)

2 The number of significant figures in 20.034 is:

A 2  
B 3  
C 4  
D 5  
E 6

3 The solution to 1303.45 + 23000 with the correct number of significant figures is:

A 24303.5  
B 24303.50  
C 24300  
D 24303  
E 24000

4 The solution to 25.69 \( \times \) 2.5040 with the correct number of significant figures is:

A 64.3  
B 64.33  
C 64  
D 64.328  
E 64.3278

5 If \( A = \frac{2B+3}{4} \) is transposed to make \( B \) the subject, then:

A \( B = 2A - \frac{3}{2} \)  
B \( B = \frac{4A+3}{2} \)  
C \( B = 2A - 3 \)  
D \( B = 4A - \frac{3}{2} \)  
E \( B = 4A + \frac{3}{2} \)

Questions 6 and 7 refer to the shape at right.

6 Using \( \pi = \frac{22}{7} \), the perimeter of a certain shape is given by

\( P = x + x + x + \frac{11x}{7} \). When transposed to make \( x \) the subject, \( x \) is:

A \( \frac{7P}{32} \)  
B \( \frac{32}{7P} \)  
C \( 7P - 14 \)  
D \( \frac{7P}{14} \)  
E \( \frac{7(P-3)}{11} \)

7 If the perimeter of the above shape is 8 cm, then \( x \) is equal to:

A \( \frac{4}{7} \) cm  
B 42 cm  
C 4 cm  
D \( \frac{32}{11} \) cm  
E 1.75 cm

8 The solution to the equation \( \frac{2x}{3} - 5 = -1 \) is:

A 1  
B 2  
C 3  
D 5  
E 6

9 To solve \( 12 - \frac{3x}{4} = 6 \), the following operations could be performed to both sides of the equation:

A Add 12, multiply by 4, divide by \(-3\)  
B Multiply by 4, divide by \(-3\), subtract 12  
C Multiply by \(-4\), divide by 3, subtract 12  
D Subtract 12, multiply by 4, divide by \(-3\)  
E Multiply by 4, subtract 12, divide by \(-3\)

10 An equation that is the same as \( 2(3x - 1) = 5x + 3 \) is:

A \( 6x = 5x + 1 \)  
B \( 11x = 5 \)  
C \( x - 2 = 3 \)  
D \( -2 = 11x + 3 \)  
E \( 11x - 2 = 3 \)
11 The value for $x$ that satisfies the equation \( \frac{1-2x}{3} + \frac{x}{4} = 2 \) is:

A 4  B 3  C \( \frac{3}{4} \)

D -4  E -3

12 The value for $x$ that satisfies the equation \( \frac{6}{x+1} - \frac{8}{x} = \frac{4}{x} \) is:

A 12  B 6  C 2  D -2  E -6

13 The sum of solutions of the pair of simultaneous equations $y + x = 12$ and $2y - x = 6$ is:

A 36  B 24  C 20  D 18  E 18

14 If $y = 3x - 4$ and $y = 5 + 4x$, then the values of $x$ and $y$, respectively, are:

A $-9$ and $-31$  B $9$ and $31$  C $-31$ and $-9$  D $-9$ and $31$  E $9$ and $-31$

15 The point of intersection of the two lines in the graph at right is:

A $(1, 3)$  B $(\frac{1}{5}, \frac{3}{2})$  C $(\frac{1}{5}, \frac{3}{5})$

D $(\frac{1}{5}, \frac{3}{5})$  E $(2, 3)$

16 Which statement below is not true for the pair of simultaneous equations $y + x = 22$ and $3x - y = 26$?

A The sum of the numbers is 22.

B Three times the first number is 26 larger than the second number.

C Three times one number is 26 smaller than the other number.

D The difference between 3 times one number and the other is 26.

E When one number is subtracted from 22, the other number is obtained.

17 If 3 times a number subtracted from 6 gives 9, then the number is:

A 5  B -1  C 1  D $\frac{1}{3}$  E 3

18 The perimeter of a regular hexagon is 12.6 cm more than the perimeter of a square with the same side length. The length of the side of a hexagon is:

A 2.1 cm  B 3.15 cm  C 1.26 cm  D 12.6 cm  E 6.3 cm

19 When half a number is subtracted from 8, the result is the same as adding double that number to 2. The equation that matches this information is:

A $2x - 8 = \frac{x}{2} + 2$  B $8 - 2x = 2 + \frac{x}{2}$  C $8 - \frac{x}{2} = 2 + 2x$

D $\frac{x}{2} + 8 = 2x + 2$  E $\frac{x}{2} - 8 = 2x + 2$

20 The sum of two numbers is 42 and their difference is 4. The smaller of the numbers is:

A 23  B 17  C 18  D 19  E 24

21 Ben is 1 year short of being twice as old as Esther. If their ages total 20 years, Ben is:

A 11  B 12  C 13  D 14  E 15

22 When \( \frac{5(x-1)}{3} - \frac{(x+3)}{2x} \) is expressed as \( \frac{ax^2 + bx + c}{6x} \), then $a$, $b$ and $c$, respectively, are:

A 10, 13 and 9  B 10, -13 and 9  C -10, 13 and -9  D 10, -13 and -9  E 10, 13 and 9

23 Simplifying \( \frac{9-e^2}{e^2} + \frac{2(e+3)}{e^3} \) gives:

A $\frac{3-e}{2}$  B $\frac{3-e}{2e}$  C $\frac{e(3-e)}{2}$  D $\frac{3+e}{2}$  E $\frac{3+e}{2e}$
24 The solution to \( x - \frac{mx}{2} = x - 3p \) in terms of \( x \) is:

A \( \frac{6}{m} \)  \hspace{1cm} B \( \frac{6p}{m} \)  \hspace{1cm} C \( \frac{-6}{m} \)

D \( \frac{-6p}{m} \)  \hspace{1cm} E \( \frac{6}{p} \)

1 Adrian has begun a new job as a car salesperson. His fortnightly wage is calculated in two parts: a set amount of $600, plus 2% of sales made each fortnight.

a Write the rule describing Adrian’s fortnightly wage.

b How much can Adrian expect to earn if his sales in a particular fortnight are:

i $20 000?
ii $65 000?
iii $100 000?

c How much must Adrian make in sales to obtain a fortnightly wage of:

i $1300?
ii $1800?
iii $2400?

Brett, also a salesperson in the motor vehicle industry, is paid a fortnightly salary of $860 regardless of sales made.

d Compare Adrian’s fortnightly wage to Brett’s fortnightly salary.

e Write the rule describing Brett’s fortnightly salary.

f How much would Adrian have to make in sales in one fortnight to obtain the same amount as Brett earns?

2 Joseph has $15 000 to invest. He does not want to ‘keep all of his eggs in the one basket’, so he decides to split the money in the following ways. He puts some of his money in the bank, which offers an interest rate of 6% p.a., and the remainder into a building society, which offers an interest rate of 11% p.a. If Joseph plans to take a trip to Queensland costing $1200, and he wants to pay for the trip using only the interest earned from his investments after 1 year, how must he split his $15 000?

3 Michael wishes to rent a car for a long weekend. The cost, \( C \), of renting a Toyota Corolla from company A is given by \( C = 25 + 0.08n \), and the cost of renting from company B is given by \( C = 40 + 0.05n \), where \( n \) is the number of kilometres travelled.

\[ C \]
\[ \text{Cost ($)} \]
\[ 0 \rightarrow 25 \rightarrow 40 \]
\[ \text{Number of km} \]
\[ n \]

[1]  \hspace{1cm} [2]

a Which company, A or B, does line [1] represent?

b What could the numbers 25 and 40 represent?

c What does the point of intersection of lines [1] and [2] represent?

d Find the coordinates of the point of intersection.

e If Michael decides to travel along the Great Ocean Road, which is about 350 km each way, from which company, A or B, should he rent so that he pays less?

f Next long weekend, Michael is planning to go to Phillip Island, which is about 150 km each way. From which company should he rent this time?

g Explain to Michael how he can decide from which company to rent, if he knows the approximate distance he intends to travel, without doing any calculations.
h Write the formula for $d$, the difference between the cost of renting the car from the two companies (A or B).

i Write the difference equation that corresponds to the equation in part h.

j Use the difference equation to generate a table of values for distances from 0 to 1000 km inclusive, with increments of 100 km. Hence, find the distance for which the cost of renting from company A will exceed the cost of renting from company B by more than $10.

**4** Novak Novelties manufactures a variety of children’s 3-D puzzles. The director of the company has asked his assistants Caitlin, Bridget and Emese to prepare a report on production costs, expenses and returns on the puzzles. Each puzzle costs the company $15 to produce. In addition, the company has monthly overheads of $21 000. The selling price of each puzzle is $45.

a Write an equation describing the expenses; that is, the total cost, $C$, of producing $n$ puzzles each month.

b Write an equation describing the selling price of $n$ puzzles.

c Plot and label the graph of the equation obtained in part a. Does it commence at the origin? Explain.

d Plot and label the graph of the equation obtained in part b on the same axis. Does it commence at the origin? Explain.

e The point of intersection of the two lines on your graph is called the break-even point. Explain what this means in terms of the given problem.

f Find the coordinates of the break-even point (point of intersection).

g Shade the portion between the two lines to the left of the break-even point. Explain what this portion represents.

h Shade the portion between the two lines to the right of the break-even point. Explain what this portion represents.

Profit may be defined as the selling price minus the total cost.

i Write an equation describing the profit obtained, $P$, after selling $n$ puzzles.

j Determine whether a profit or loss is made when:

i 400  
ii 600  
iii 800  
iv 1000 puzzles are sold in a particular month.
Chapter opener

**DIGITAL DOC**
- 10 Quick Questions doc-9883: Warm up with ten quick questions on algebra. (page 87)

**4A Review of index laws**

**TUTORIAL**
- WE3 eles-1480: Watch how to simplify expressions involving indices. (page 88)

**4B Standard form and significant figures**

**TUTORIAL**
- WE5 eles-1481: Watch how to simplify an expression involving the product and quotient of numbers in standard form. (page 91)

**4C Transposition**

**TUTORIAL**
- WE9 eles-1482: Watch how to solve equations for specific pronumerals. (page 94)

**DIGITAL DOC**
- WorkSHEET 4.1 doc-9965: Transpose simple equations, and use transposition to solve worded problems. (page 97)

**4D Solving linear equations and simultaneous linear equations**

**TUTORIAL**
- WE14 eles-1483: Watch how to solve equations involving fractions. (page 100)

**DIGITAL DOC**
- Investigation doc-9884: Comparing production costs. (page 107)

**4E Applications**

**DIGITAL DOC**
- WorkSHEET 4.2 doc-9966: Transpose equations with algebraic fractions and apply this method to more complex worded problems. (page 114)

**4F Algebraic fractions**

**TUTORIAL**
- WE25 eles-1484: Watch how to simplify algebraic expressions involving fractions. (page 115)

**4G Linear literal equations**

**TUTORIAL**
- WE28 eles-1485: Watch how to solve literal simultaneous equations. (page 118)

Chapter review

**DIGITAL DOC**
- Test Yourself doc-9885: Take the end-of-chapter test to test your progress. (page 126)

To access eBookPLUS activities, log on to www.jacplus.com.au
**ALGEBRA**

**Exercise 4A**

**Review of index laws**

1. (a) $4ab$
   (b) $4a^2b^3$
   (c) $\frac{a^2}{2b^3}$
   (d) $\frac{a^5b}{2}$
   (e) $\frac{7}{6a^2}$
   (f) $\sqrt[a]{a^2b}$
   (g) $\frac{1}{2}$
   (h) $\frac{y^3}{4x^3}$
   (i) $\frac{1}{2}$
   (j) $\frac{1}{2n}$
   (k) $\frac{25m^9}{9a^6}$
   (l) $\frac{-1}{2888v^{13}w^6}$

3. (a) $2^{3n+1}$
   (b) $3^{2n+3}$
   (c) $3 \times 5^n - 3$
   (d) $\frac{3^{2n}}{2^{n+8}}$
   (e) $3^{2n}$
   (f) $3^{4n-4} \times 2^{3n-4}$
   (g) $\frac{3}{33}$
   (h) $\frac{5}{2}$
   (i) $\frac{7}{x^6}$
   (j) $\frac{5}{2}$
   (k) $\frac{7}{x^6}$
   (l) $\frac{3}{2}$

**Exercise 4B**

**Standard form and significant figures**

1. (a) $3.604 \times 10^8$
   (b) $2.13457 \times 10^7$
   (c) $1.02939 \times 10^3$
   (d) $3.24 \times 10^2$
   (e) $1.0031 \times 10^{-4}$
   (f) $5.70201009 \times 10^8$
   (g) $10000$
   (h) $5000$
   (i) $0.00008$
   (j) $0.0003$
   (k) $0.016$
   (l) $0.4$
   (m) $5 \times 10^7$
   (n) $5 \times 10^-10$
   (o) $0.09$
   (p) $0.245$
   (q) $1.5$
   (r) $16$

3. (a) $2^5$
   (b) $2^4$
   (c) $2^3$
   (d) $2^2$
   (e) $2^1$
   (f) $2^0$
   (g) $2^{-1}$
   (h) $2^{-2}$
   (i) $2^{-3}$
   (j) $2^{-4}$

**Exercise 4C**

**Transposition**

1. (a) $x = \frac{5}{2}y$  $y = \frac{4}{5}x + 4$
   (b) $y = \frac{3}{4}x + 3$
   (c) $a = \frac{1}{2}m + \frac{14}{5}$
   (d) $k = \frac{3}{2} - \frac{3}{y}$
   (e) $a = \frac{3}{2}b$ or $b = \frac{2}{a}$
   (f) $a = 2 + \frac{1}{5}b$ or $b = 5a - 10$
   (g) $c = -2a + 6b$
   (h) $a = \frac{5}{2}b + 3$

2. (a) $\frac{v^2 - a^2}{s}$, $u = \pm \sqrt{v^2 - as}$
   (b) $r = \pm \frac{\sqrt{s}}{\sqrt[4]{4\pi}}$
   (c) $R = \frac{R_0R_2}{R_1R_2}$, $R_1 = \frac{R_0R_2}{R_2 - R}$
   (d) $100 \left( \frac{A}{A_0} - 1 \right)$
   (e) $t = \frac{2x}{u + v}$, $u = \frac{2x}{t - v}$
   (f) $L = \frac{g(T)}{2\pi}$, $g = L \left( \frac{2\pi}{T} \right)^2$
   (g) $b = \pm \sqrt{C^2 - a^2}$
   (h) $a = \frac{2(s - ut)}{i^2}$
   (i) $I = \pm \frac{P}{\sqrt{R}}$, $R = \frac{P}{\sqrt{T}}$
   (j) $m = \frac{vR}{v_2 - v_1}$, $v_1 = v_2 - \frac{vR}{m}$

3. (a) 3
   (b) 4
   (c) 5
   (d) 6
   (e) 7
   (f) 8
   (g) 9
   (h) 10

11. $n = \frac{S}{180} + 2$

<table>
<thead>
<tr>
<th>Polygon</th>
<th>Number of sides $(n)$</th>
<th>Sum of interior angles $(S)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangle</td>
<td>3</td>
<td>180°</td>
</tr>
<tr>
<td>Hexagon</td>
<td>6</td>
<td>720°</td>
</tr>
<tr>
<td>Dodecagon</td>
<td>12</td>
<td>1800°</td>
</tr>
<tr>
<td>Nonagon</td>
<td>9</td>
<td>1260°</td>
</tr>
<tr>
<td>Heptagon</td>
<td>7</td>
<td>900°</td>
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<td>Octagon</td>
<td>8</td>
<td>1080°</td>
</tr>
<tr>
<td>Pentagon</td>
<td>5</td>
<td>540°</td>
</tr>
<tr>
<td>Quadrilateral</td>
<td>4</td>
<td>360°</td>
</tr>
<tr>
<td>Decagon</td>
<td>10</td>
<td>1440°</td>
</tr>
</tbody>
</table>

**Exercise 4D**

**Solving linear equations and simultaneous linear equations**

1. (a) $-19$
   (b) $\frac{7}{2}$
   (c) $-50$
   (d) $5\frac{5}{6}$
   (e) $2$
   (f) $12$
   (g) $25$
   (h) $44$
   (i) $-59$
   (j) $3$
   (k) $-18$
   (l) $4\frac{1}{2}$
   (m) $-1$
   (n) $7$
   (o) $1$
   (p) $2$
   (q) $7$

2. (a) $-1$
   (b) $1$
   (c) $2$
   (d) $-5$
   (e) $2$
   (f) $0$
   (g) $2$
   (h) $6$
   (i) $2$
   (j) $-3$
   (k) $\frac{1}{2}$
   (l) $24$
   (m) $-3$
   (n) $2$
   (o) $0$
   (p) $\frac{7}{17}$
   (q) $4$
   (r) $\frac{1}{2}$
   (s) $2$
   (t) $\frac{7}{17}$
   (u) $2$

3. (a) $-1$
   (b) $1$
   (c) $2$
   (d) $-2$
   (e) $7$
   (f) $3$
   (g) $4$
   (h) $-14$
   (i) $\frac{1}{2}$
   (j) $\frac{13}{47}$
   (k) $\frac{2}{32}$
   (l) $19$
   (m) $27$

4. (a) $2$
   (b) $3$
   (c) $2$
   (d) $-2$
   (e) $7$
   (f) $3$
   (g) $\frac{7}{17}$
   (h) $3$

5. (a) $10^0 - 1$
   (b) $10^1 - 1$
   (c) $10^2 - 1$
   (d) $10^3 - 1$
   (e) $10^4 - 1$
   (f) $10^5 - 1$

6. (a) $1$
   (b) $2$
   (c) $3$
   (d) $4$
   (e) $5$
   (f) $6$
   (g) $7$
   (h) $8$
   (i) $9$
   (j) $10$
   (k) $11$
   (l) $12$
   (m) $13$
   (n) $14$
   (o) $15$
   (p) $16$

7. (a) $2$
   (b) $3$
   (c) $4$
   (d) $5$
   (e) $6$
   (f) $7$
   (g) $8$
   (h) $9$

8. (a) $0$
   (b) $1$
   (c) $2$
   (d) $3$
   (e) $4$
   (f) $5$
   (g) $6$
   (h) $7$

9. (a) $0$
   (b) $1$
   (c) $2$
   (d) $3$
   (e) $4$
   (f) $5$
   (g) $6$
   (h) $7$

10. (a) $-1$
    (b) $-2$
    (c) $-3$
    (d) $-4$
    (e) $-5$
    (f) $-6$
    (g) $-7$
    (h) $-8$

11. (a) $-1$
    (b) $-2$
    (c) $-3$
    (d) $-4$
    (e) $-5$
    (f) $-6$
    (g) $-7$
    (h) $-8$

**Exercise 4E**

**Applications**

1. $x = 1$, $y = -2$, $z = 3$

2. $t = 3$

3. $60$ or $1\frac{1}{2}$

4. $18$ red tulips, $10$ yellow tulips

5. Width = $8$ cm, length = $20$ cm

6. $10$ cm, $10$ cm, $12$ cm

7. $40^\circ$, $120^\circ$, $20^\circ$, obtuse angled
8 S99
9 a B b D
10 a 10 b 15
11 a 3
   b 3 years in America, 4 years in Germany,
   8 years in London, 1 year in Australia
12 15 Jonathan, 20 Golden Delicious
13 Alex will reach Nat in 1.25 h; that is, at
   11.30 am.
14 4 km/h
15 10 h
16 12 days, No
17 19, 4
18 20, 7
19 17, 5
20 $1 = 7 cm
   w = 5 cm
   A = 35 cm²
21 x = 8 cm
   y = 10 cm
   P = 50 cm
22 33 cm
23 35 cm
24 6 cm
25 6 cm
26 a

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b 5 cm × 5 cm; square
27 54; Yes
   18; No
28 Toaster: $19.95
   Sandwich maker: $24.95
29 Pool and spa: 120
   Pool, spa, sauna and steam room: 73
30 5c coins: 34
   10c coins: 8
31 Lamb: $2.50
   Pork: $4.00
32 23
33 Dim sims: 5
   Spring rolls: 4

Exercise 4G Linear literal equations
1 a x = \( \frac{bc}{a} \)
   b x = \( \frac{y \cdot w}{2} \)
   c x = \( \frac{mp}{2} + m \)
   d x = 3st - r
   e x = \( \frac{d}{f + g} \)
   f x = \( \frac{3k}{4l - l} \)
   g x = \( \frac{a}{ac} \)
   h x = \( \frac{n}{m(n + 1)} \)
   i x = b + \( \frac{a - b}{a} \)
   j x = \( \frac{a}{b} + s \)
   k x = \( \frac{1}{br} + s \)
   l x = q - n
   m x = \( \frac{1}{ef} - d \)
   n x = \( \frac{n^3}{mb + cn} \)
   o x = \( \frac{d}{3 - 2c} \)
   p x = \( \frac{ef}{d} \)
2 D
3 a x = a, y = b
   b x = \( a - b \)
   c x = \( \frac{2a}{b} \)
   d x = \( a + b \)
   e x = \( \frac{a}{b} \)
   f x = \( \frac{1}{a - b} \)
   g x = \( \frac{b - a}{2b} \)
   h x = \( \frac{a - 2b}{2b} \)
4 a 4
   b - 2

Exercise 4D Algebraic fractions
1 a \( \frac{25a}{28} \)
   b \( \frac{23b}{12} \)
   c \( \frac{-17d}{15} \)
   d \( \frac{g}{6} \)
   e \( \frac{h^2 + 20}{5h} \)
   f \( \frac{4k + 15}{6} \)
   g \( \frac{m^2 - 14}{7m} \)
   h \( \frac{1 - 2n^2}{3n} \)
2 a \( \frac{5p + 4}{p(p + 2)} \)
   b \( \frac{13q + 15}{2g(q + 5)} \)
   c \( \frac{7r - 5}{(r + 1)(r - 2)} \)
   d \( \frac{-6s + 25}{(s - 3)(s + 4)} \)
   e \( \frac{3p^2 + kq}{k + p + 3} \)
   f \( \frac{3pk^2 + kw}{k + p + 3} \)
3 a x = a, y = b
   b x = \( \frac{a - b}{2b} \)
   c x = \( \frac{a - b}{2b} \)
   d x = \( \frac{a + b}{2b} \)
   e x = \( \frac{a}{b} \)
   f x = \( \frac{a}{b} \)
   g x = \( \frac{1}{a - b} \)
   h x = \( \frac{1}{2b} \)
4 a 4
   b - 2

CHAPTER REVIEW

SHORT ANSWER
1 \( 45x \frac{15}{15} \)
2 a 0.00375
   b 0.000000048
3 a 106.51
   b 1200
4 a \( x = 2y - \frac{5}{2} \)
   b \( d = \frac{-43}{21} \)
   c \( e = \frac{23}{21}d \)
5 x = \( \frac{6}{7} \)
6 a y = \( 2x + 2 \)
   b \( y = 0.5x - 4 \)
   c \( b(0.8, 3.6) \)
7 a \( 2, 0 \)
   b \( (4, 1) \)
   c \( (1, -6) \)
8 20 of 20c
   5 of 10c
   10 of 5c
9 T = 2300 + 500n - 135e
   n = \( T - 2300 + 135e \)
   e = \( \frac{2300 + 500n - T}{135} \)
10 Rebecca: 7 years old
   Jessica: 10 years old
11 a \( \frac{13m - 24}{6} \)
   b \( \frac{11x + 16}{x(3x - 4)} \)
   c \( \frac{5x - 5}{(x + 1)(x + 2)(x - 3)} \)
   d \( \frac{9(g + 3)^5}{(g - 1)^2} \)
12 \( 10(5w - 3)(w + 4) \)
13 a \( \frac{b - gmn}{a} \)
   b \( \frac{x^2 + y^2}{k + p + 3} \)
15 a \( x = 2a \)
   b \( y = -2b \)
16 \( 123 \)
17 B
   18 E
19 C
   20 D
   21 C
22 D
   23 C
   24 B

MULTIPLE CHOICE
1 B
2 D
3 E
4 B
5 A
6 A
7 E
8 E
9 D
10 C
11 D
12 D
13 B
14 A
15 D
16 C
17 B
18 E
19 C
20 D
21 C
22 D
23 C
24 B
EXTENDED RESPONSE

1. **W = 600 + 0.02s**
   - a) $1000
   - b) $1900
   - c) $2600
   - d) Brett is paid $860 each fortnight regardless of whether or not he makes any sales. However, the amount Adrian takes home depends on how many sales he can make. He will take home a minimum of $600 each fortnight but must continue to make sales if he wishes to increase this amount.
   - e) $S = 860$
   - f) $S = 13 000$

2. **9000 for 6%, 6000 for 11%**

3. a) B
   - b) Fixed fee
   - c) Same cost
   - d) $(500, 65)$
   - e) B
   - f) A
   - g) Under 500 km — choose A; over 500 km — choose B; equal to 500 km — either
   - h) $d = 0.03n - 15$

4. **C = 15n + 21 000**
   - a) Between 800 and 900
   - b) $SP = 45n$

5. **The break-even point occurs when expenses (total cost of manufacturing the puzzles) equal the selling price (money received from sale of puzzles). Therefore, the company is neither making a profit nor running at a loss.**
   - c) $700, 31 500$
   - d) Refer to the graph in part c.
   - e) Yes, the graph does commence at the origin. This occurs because if there are no sales, no money is received.
   - f) Refer to the graph in part c. This portion (the blue portion) of the graph represents expenses (total cost of manufacturing the puzzles) being greater than the selling price (money received from sale of puzzles). Therefore, the company is making a loss.
   - g) Refer to the graph in part c. This portion (the yellow portion) of the graph represents the selling price (money received from sale of puzzles) being greater than the expenses (total cost of manufacturing the puzzles). Therefore, the company is making a profit.
   - h) $P = 30n - 21 000$
   - i) $9000 (a loss of $9000)$
   - j) $3000 (a loss of $3000)$
   - k) $3000 (a profit of $3000)$
   - l) $9000 (a profit of $9000)$